Roll No. ....

Y – 3187 (A)

M.A./M.Sc. (Mathematics) (Fourth Semester) (SPECIAL)

EXAMINATION, August 2021

(SECOND CHANCE)

Paper – 412

SPECIAL FUNCTIONS

Time : Three Hours

Maximum Marks : 85 (For Regular Students) *Minimum Pass Marks : 29*

Maximum Marks : 100 (For Private Students) *Minimum Pass Marks : 34*

Note—Attempt *all* questions.

1. Attempt *all* parts— 17/20
   (i) Establish the relation between Beta and Gamma functions
   (ii) Define Hyper geometric functions
   (iii) Define Legendre function
   (iv) Define Laguerre Polynomials
   (v) Define Macrobert's E-Function.

2. Given the Weierstrass and Euler's definitions of $\lceil z$ and establish their equivalence by following inequality. 17/20

$$0 \le e^{-t} - \left(1 - \frac{t}{n}\right)^n \le t^2 \frac{e^{-t}}{n}$$

3. Define Bessel's function $J_n(z)$ and show that— 17/20

$$J_{-n}(z) = (-1)^n J_n(z)$$

4. Define Laguerre polynomials and show that— 17/20

$$\lceil(1-\propto)(xt)^{-\frac{\propto}{2}} e^t J_\propto \left(2\sqrt{xt}\right) = \frac{\sum_{n=0}^{\infty} \angle_n^{(\propto)}(x) t^n}{(\pi \propto)_n}$$

5. Prove that— 17/20

(a) $$G_{pq}^{mn} = \left(x^{-1} \Big/ \begin{matrix} ar \\ bs \end{matrix}\right) = G_{qp}^{nm} \left(x \Big/ \begin{matrix} 1-bs \\ 1-ar \end{matrix}\right)$$

(b) Prove that—

$$G_{02}^{10} = (x / a, b) = x^{\frac{1}{2}(a+b)} \sqrt{a-b}\left(2x^{1/2}\right)$$