Roll No. ....

Note : Attempt *all* questions.

1. Prove that if g is continuous on D, then the successive approximation $\{u_n(t)\}$ defined by Picard method exist as continuous function on J and $(t, u_n(t)) \in$ D for $t \in$ J.   17

2. Let $v, w \in$ C[J$_1$, R] and satisfy the inequalities :   17

    $D\_v(t) \le g(t, v\ (t))$ and
    $D\_w(t) > g(t, w(t))$ with

    $(t, v(t)), (t, w(t)) \in \Omega$ for $t \in$ J$_1$ then prove that :
    $v(t_0) < w(t_0)$ implies $v(t) < w(t), t \in$ J$_1$.

3. Show that the system of equations $\dfrac{dx}{dt} = -2x + 3y + xy, \dfrac{dy}{dt} = -x + y - 2xy^2$

    is almost linear and discuss the type of stability of critical point (0, 0).   17

4. Discuss the stability of critical point of the following two-dimensional system and draw the phase diagram :   17

$$x_1' = -x_1 + 2x_2, \ x_2' = -2x_1 - 5x_2.$$

5. Show that in the vector field :   17

$$\dot{x} = f(x, \mu) = \mu x - x^3, x \in R', \mu \in R'$$

    the supercritical pitchfork bifurcation occurs.