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W-3318

M.A./M.Sc. (Fourth Semester) Examination, June-2020

MATHEMATICS

Paper - 412

Special Functions

Time : Three Hours

Maximum Marks : 85 (For Regular)
Minimum Pass Marks : 29
Maximum Marks : 100 (For Private)
Minimum Pass Marks : 34

Note : Attempt **all** questions.

Unit-I

Q.1. a) For a complex number Z (Z not an integer), Show that $\Gamma(Z)\Gamma(1-Z)=\dfrac{\Pi}{Sim\Pi Z}$.

b) State and prove Gauss multiplication theorem.

Unit-II

Q.2. a) If Re(*c*–*a*–*b*)>0 and *c* is neither zero nor a negative integer, then prove that-

$$F(a,b;c;1)=\frac{\Gamma(c)\ \Gamma(c-a-b)}{\Gamma(c-a)\ \Gamma(c-b)}$$

b) Prove that $\dfrac{d^n}{dx^n}\left[x^{a-1+n}F(a,\ b;c;x)\right]=(a)_n\ x^{a-1}F\begin{bmatrix}a+n,\ b; & x\\ c; & \end{bmatrix}$

Unit-III

Q.3. Define Bessel function. Prove that when *n* is a positive integer, $J_n(x)$ is the coeffiecinet of t^n in the expansion of $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)}$ in the ascending and descending powers of t.

Unit-IV

Q.4. a) Show that $\int\limits_{-\infty}^{\infty} x^2e^{-x}\{H_n(x)\}^2\,dx=\sqrt{\pi}\,2^n\,n!\left(n+\dfrac{1}{2}\right)$

b) Prove that $H_n^1(x)=2n\ H_{n-1}(x)\ for\ n\geq 1$

Unit-V

Q.5. a) Show that

$$(\rho_1-1)xE(\alpha_1,..........,\alpha_p;\rho_1,..........,\rho_q:x)=xE(\alpha_1,..........,\alpha_p;\rho_1-1,\rho_2,..........,\rho_q:x)$$
$$+E(\alpha_1+1,..........,\alpha_p+1;\ \rho_1+1,..........,\rho_q+1:x)$$

b) Prove that if $\operatorname{Re}(x)>0,\operatorname{Re}(\gamma)>0,$ then

$$\int\limits_0^{\infty} e^{-x\lambda}\,\lambda^{\gamma-1}\,{}_2F_1\begin{pmatrix}\alpha,\beta; & -\lambda\\ \delta; & \end{pmatrix}d\lambda=\frac{\Gamma(\delta)\ x^{-\gamma}}{\Gamma(\alpha)\ \Gamma(\beta)}E(\alpha,\beta,\gamma;\delta;x)$$