NUCLEAR PHYSICS-401

NUCLEAR DECAY

UNIT- III

Topics

Fermi Kurie Plot, Selection Rule, Allowed and Forbidden Transition, Parity in Beta Decay, Gamma Transition, Selection Rule, Internal Conversion, Recoil Nucleus and Nuclear Isomerism

Then (19) became

$$N(K) = C_1\sqrt{K(K+2m_ec^2)}\,(Q-K)^2(K+m_ec^2)\,F(Z_d,K) \quad (21)$$

Equation (21) shows perfect matching between the theory and the experimental data.

~~SELECTION RULE~~ FERMI KURIE PLOT

Writting the (17) in momentum distribution from we have (for electron)

$$N(P_e)dP_e = C_0 P_e^2 P_\nu^2 \, dP_e \; F(Z_d,K) \quad (22)$$

$$\Rightarrow \sqrt{\frac{N(P)}{P_e^2 F(Z_d,K)}} \propto P_\nu \quad (23)$$

We know that $P_\nu \propto Q-K$ and putting in (23)

$$\Rightarrow \sqrt{\frac{N(P)}{P_e^2 F(Z_d,K)}} \propto Q-K \quad (24)$$

Equation (24) is known as Fermi ~~Kurie~~ Kurie plot. This was done show as to make the Fermi function linear. Fermi Kurie is a standard way to check wheather this decay can be described by Fermi theory.

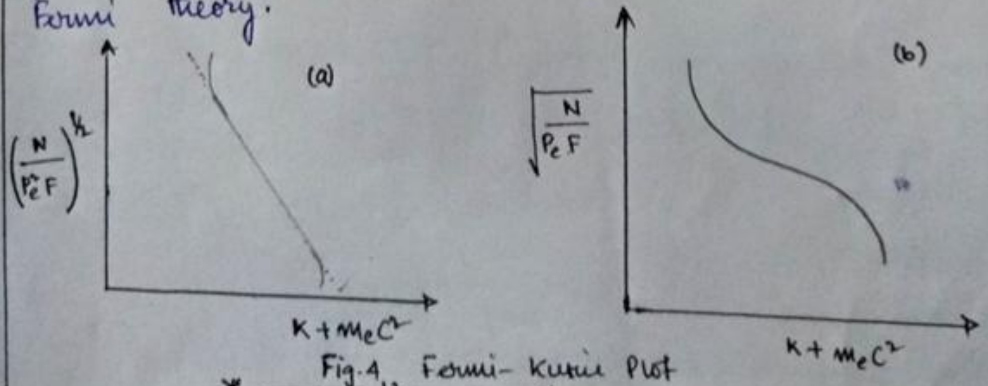


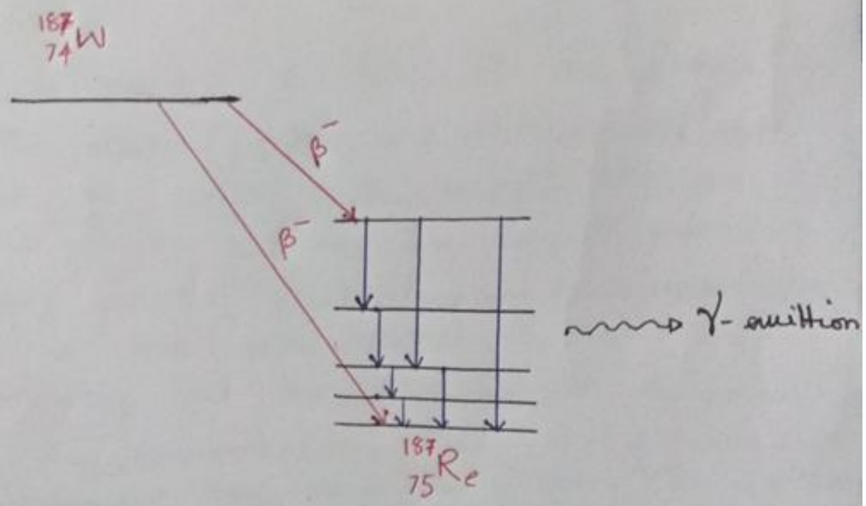
Fig. 4 Fermi- Kurie Plot

If the Fermi-Kurie plot is extrapolated it give the end point energy of the β-decay. The graphs in Fig 4 shows the deviation from the straight line at the lower energy. This deviation may be due to the following reasons,

i) Scattering of β- particles inside the nuclei during the radio-active decay.

ii) Forbidden transition occur during β-decay.

iii) Transition to two or more states of daughter nucleus (Fig 5)



SELECTION RULE, ALLOW AND FORBIDDEN TRANSITION

Let us consider the β^+-decay of $^{66}_{31}Ga$ and $^{91}_{39}Y$

$$^{66}_{31}Ga \longrightarrow {}^{66}_{30}Zn + e^+ + \nu \qquad (25)$$

Here both $^{66}_{31}Ga$ and $^{66}_{30}Zn$ (parent and daughter) have the same spin parity of 0^+. Thus it match with the Fermi expression.

$$^{91}_{39}Y \longrightarrow {}^{91}_{38}Zr + e^+ + \nu \qquad (26)$$

Here the spin parity of parent $^{91}_{39}Y$ is $\frac{1}{2}^-$ and daughter $^{91}_{38}Zr$ is $\frac{5}{2}^+$. Thus the Fermi expression doesnot match will with the experimental data.

Considering the general ~~form from~~ form of angular momentum parity of the parent and daughter nucleus

$$(I^{\pi})_p \longrightarrow (I^{\pi})_d + I(e+\nu) \qquad (27)$$

$I(e+\nu)$ represent the out going particle.

From the previous equations (9A), (9B), (10A) and (10B) we have

$$\psi_e = \frac{1}{\sqrt{V}} \qquad (28)$$

The above wave function is taken at the origine of the nucleus. The electron/positron and neutrino/anti neutrino are created at the center of the nucleus and coming out perpendicular distance to the radius. The angular momentum of the out going particle (electron/positron ~~or~~ neutrino/anti-neutrino) will be zero with respect to the daughter nucleus (approximating that the wave function of electron and neutrino are constant), $l=0$. But as electron and neutrino are fermions they have the $\frac{1}{2}$ spin. The combination both the spin value may have, $S = 0, 1$.

1. consider

$l=0$, $S0$, then $J=0$ [for out going particle]

Thus the spin of angular momentum for both parent and daughter nuclei have same value. In the above case the parity of the system docent change. This ~~lead~~ lead to $\Delta I=0$ and $\Delta\pi=0$ such transitions are known as Allowed Fermi transition

2. If $l=0$, $S=1$ and $J=1$

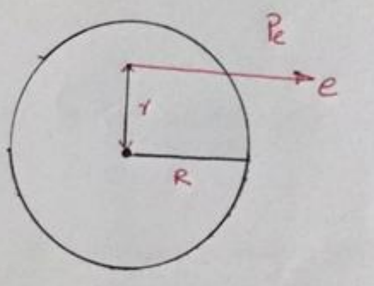
In this case the change in angular momentum occur from $(I^{\pi})_p$ to $(I^{\pi})_d$ will have one unit more or less ie

$$(\vec{1}^{\lambda})_p \longrightarrow (\vec{1}^{\lambda}_{d})_{d+1}$$

Thus $\Delta I = 0, 1, -1$ and $\Delta \lambda = 0$

This types of transition is called Allowed Gamma Teller transition.

3. Now if $L \neq 0$ and check the possibility of the case considering that the shape of the nucleus is perfect spherical and electron is emitted from the distance (perpendicular distance) r from the origin with linear momentum P_e



The angular momentum = $P_e r$

The maximum angular momentum of the emitted electron = $P_e R$ (emitted from the surface of the nucleus).

To calculate the linear momentum of an electron. Let us consider that Kinetic energy (KE) of β-particle = 1 MeV

The relativistic expression for energy momentum relation is

$$(P_e^2 c^2 + m_e^2 c^4)^{1/2} = KE + m_e c^2$$

$$\Rightarrow P_e^2 c^2 + m_e^2 c^4 = (KE + m_e c^2)^2 \qquad (29)$$

The rest mass energy $(m_e c^2) = 0.5$ MeV

$$P_e^2 c^2 + (0.5)^2 = (1 + 0.5)^2$$

$$\Rightarrow P_e^2 c^2 = 2 \ (MeV)^2$$

$$\Rightarrow P_e = 1.4 \ \frac{MeV}{c} \qquad (29A)$$

considering the average diameter of the nucleus to be 5 fm.

The maximum angular momentum = $\frac{1.4 \times 5}{c}$

comparing the above value with quantum mechanical angular momentum

$$\sqrt{l(l+1)\hbar^2} = \frac{7.}{c} \text{ MeV fm}$$

$$\Rightarrow \sqrt{l(l+1)} = \frac{7}{c\hbar}$$

$$\Rightarrow l(l+1) = (0.03)^2$$

$$\Rightarrow l << 1 \qquad (29B)$$

This proved that there is less probability for electron neutrino to have $l \neq 0$.

And if $l \neq 0$ and still the β-decay takes places those β-decay are known as forbidden transition.

Thus if $l=1$ → first forbidden transition

$l=2$ → second forbidden transition

$l=3$ → Third forbidden transition

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$l=n$ → n^{th} forbidden transition

The selection rules for first forbidden transition

$l=1$, $\Delta\pi$ = Yes (change from parent to daughter)

$S=0$, $J=1$

$\Delta I = 0, \pm 1$

This is called first forbidden Fermi transition

If $l = \cancel{0}1, \; \Delta\pi = \text{Yes}$

$s = 1, \; J = 2, 1, 0$

$\Delta I = 0, \pm 1, \pm 2$

This types of transition are called First Forbidden Gamma Teller Transition.

Comparative half life : It is denoted by $fT_{1/2}$ and given by

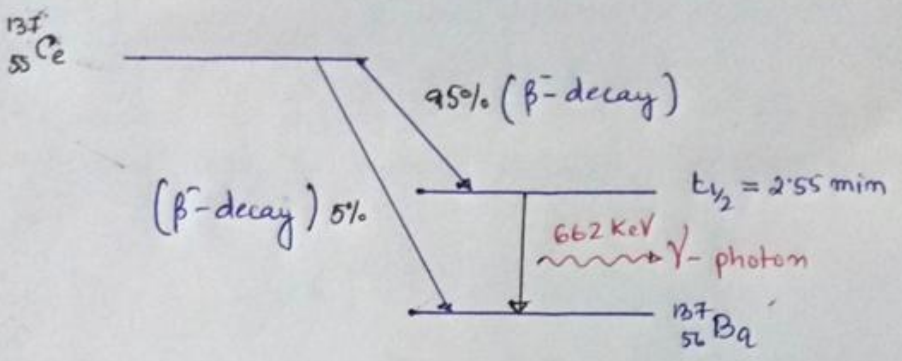
$$fT_{1/2} = \frac{2\pi^3 \hbar^7}{m_e^5 g^2 c^4} \frac{\log_e 2}{|M_{if}|^2}$$

$$= \frac{6932_0}{|M_{if}|^2} \qquad (30)$$

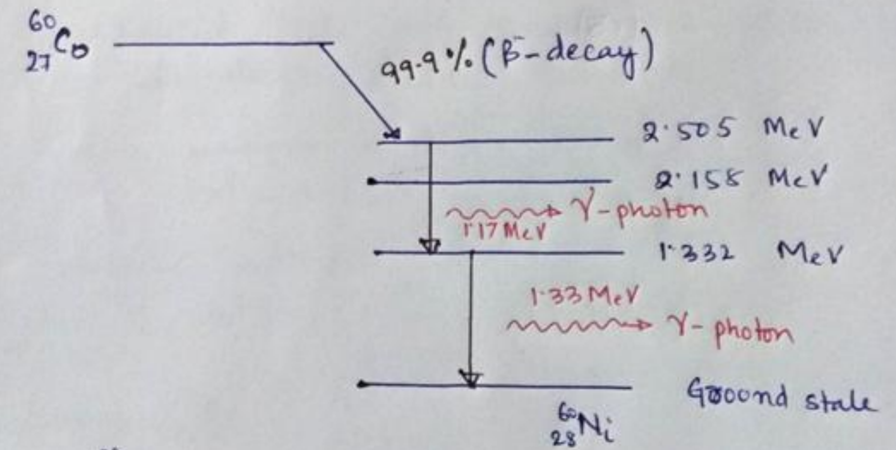
GAMMA DECAY

In case of α- and β- decay the ~~nucl nucleons~~ nucleus are changed from parent to daughter, but in case of γ- decay it doesnot change. The ~~nucleons~~ nucleus took transition from excited state to lower excited state or ground state by releasing the Gamma photon (γ- photon). γ- decay occured only after α- or β- decay which means that the daughter nucleus after the α- or β- decay in at the excited state. Let us look some of the examples in case of ${}^{137}_{55}Cs$ to ${}^{137}_{56}Ba$, ${}^{60}_{27}Co$ to ${}^{60}_{28}Ni$ in case β^-- decay and ${}^{22}_{11}Na$ to ${}^{22}_{10}Ne$ in β^+ decay or electron capture.

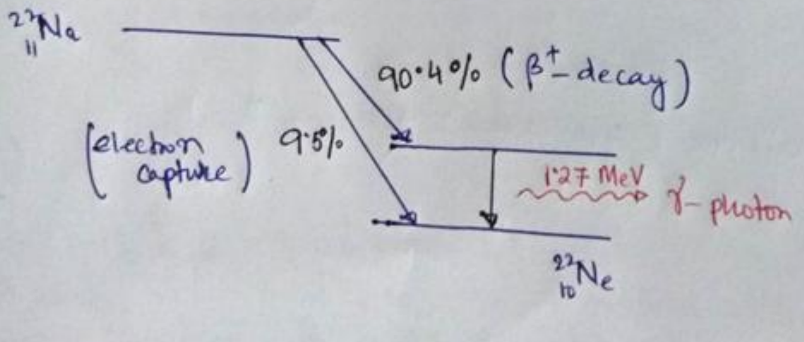
$^{137}_{55}Ce \longrightarrow ^{137}_{56}Ba$



$^{60}_{27}Co \longrightarrow ^{60}_{28}N$



$^{22}_{11}Na \longrightarrow ^{22}_{10}Ne$



The γ-decay has similar phenomenon to electron transition. The emitted γ-photon alway carried the angular momentum. Thus it give rise to angular momentum selection rules. If J_i, J_f and L are initial, final nuclear spin and L photon angular momentum. Then

$$\vec{J_i} = \vec{J_f} + \vec{L}$$

$$\Rightarrow |J_i - J_f| \leq L \leq |J_i + J_f| \qquad (31)$$

Equation (31) determine the nature of electromagnetic radiation generated by the γ-photon (dipole, quadrapole octapole etc).

It also depend on the change ~~of~~ in parity between the ~~ext~~ excited state and ground state which gave rise to magnetic or electric transition.

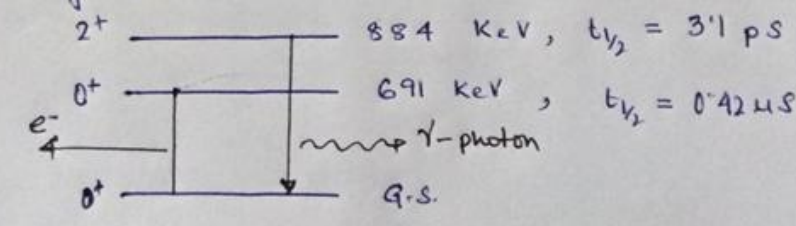
For electric ~~multipote~~ multipole transition

$$\pi^f = \pi^i (-1)^i \qquad (32A)$$

For magnetic multipole transition

$$\pi^f = \pi^i (-1)^{i+1} \qquad (32B)$$

considering $^{72}_{52}Ge$



Here $J_i = 0$, $J_f = 0$, then $L = 0$, Thus the multipolarity is zero, there is no radiation with multipolarity zero. Therefore 0^+ to 0^+ transition for gamma doesnot exist. But still there is transition takes place with $t_{1/2} =$ 0.42 μS

In this case the nucleus is deexcited and energy internally transfered to the atomic orbital electron (1s). Here the electron in 1s orbital spend for some time inside the neucleus and intereact with it (the wave function is overlap with the nuclear wave function). The intereaction between the 1s electron with neucleus at the excited state make the 1s electron to receive the energy from the nucleus. If this transfer energy is more than the electron binding energy (ionization energy) the electron will be knock of of the 1s orbital. Thus process is known as internal conversion.

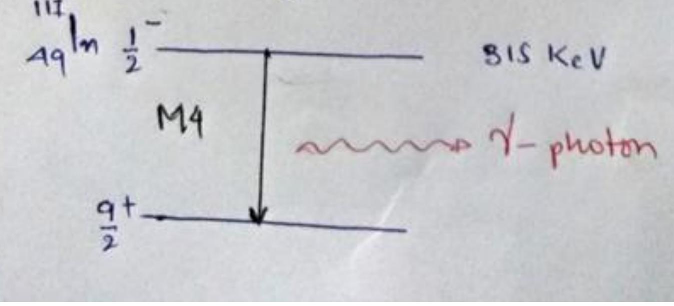
But in case of transition between 2^+ to 0^+ (G.S) the γ-photon is emitted having the L value 2 and parity with out changing. This transition is purely a quadrapole transition (E2) by multipole transition.

Note :- There is alway possible to have internal conversion alone with the γ-transition.

The ratio of probability of internal conversion to the probability of γ-decay is called conversion co-effecient.

$$\text{Conversion co-effecient} = \frac{\text{probability of intenal conversion}}{\text{probability of } \gamma\text{- decay.}}$$

Consider the $^{117}_{49}In$ γ-decay.



Here the value of L ~~are~~ is 4 or 5 as calculated from the equation (31) and parity is changed. Thus the possible transition are M4 and E5.

For electric type of transition

$$\frac{\text{probability for } (L+1)}{\text{probability for } L} \simeq 10^{-5} \qquad (33)$$

For electric magnetic type of transition

$$\frac{\text{probability for EL}}{\text{probability for ML}} \simeq 10^{2} \qquad (34)$$

In the above case M4 transition is dominating over the E5 transition. such type of transition have longer value of life time, such case are known as the isomeric state (life time interms of minute).

RESONANCE ABSORPTION

Let us consider the nucleus be at excited state with energy E_0. If γ-photon of energy E_γ is released from the ~~transition~~ nucleus during the deexcitation process leaving the final energy of nucleus E_f. According to the Newton's third law of motion the nucleus is recoil in the opposite direction with velocity v and kenetic energy K. By the conservation of momentum and energy

$$E_0 = E_\gamma + K \qquad (35A)$$

$$\frac{E_\gamma}{c} = P_{nucl} \qquad (35B)$$

$$E_0 = E_\gamma + \frac{P_{nucl}^2}{2M}$$

$$= E_\gamma + \frac{E_\gamma^2}{2MC^2}$$

$$E_\gamma = E_0 - \frac{E_\gamma^2}{2MC^2} \qquad (35c)$$

$$E_\gamma = E_0 - \frac{E_0^2}{2MC^2} \qquad (35D)$$

Since the energy diffrence is very low so $E_\gamma = E_0$ Here energy of the E_γ is less by $\frac{E_0^2}{2mc^2}$ then the original value.

If E_γ energy is ~~absorbed~~ absorbed by the same other nucleus at ground state and excited to higher energy state, then this phenomenon is called resonance absorption.

when the γ- source is placed inside the perfect crystal. It may emitt the γ- photon. when the phonon energy of the crystal is larger than the γ- energy the γ- source will perfectly emitted γ- radiation, This phenomenon is called resonance transmittion. ~~Further~~ Fulther this lead to a nuclear spectrocopy called Mössbauer spectroscopy.

REFERENCES

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