UNIT – III NUCLEAR DECAY

A beta particle is a highly enurgenetic, high-speed electron or positron emitted by the radioactive decay. Beta particle with an energy of 0.5 MeV have a range of about one meter in air.

The radioactive nuclie can emitt β- particle in three forms. They are β⁻-decay, β⁺-decay and electron-capturing. Now let us consider the three nuclear reactions

$$ {}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu} \qquad (1)$$

$$ {}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^+ + \nu \qquad (2)$$

$$ {}^{A}_{Z}X + e \longrightarrow {}^{A}_{Z-1}Y + \nu \qquad (3)$$

In the equation (1) the neutron is converting to proton, the daughter nucleus Y has one more proton than the parent nucleus. In this reaction the parent nucleus undergo the β⁻-decay by releasing the anti-neutrino.

The equation (2) shows the conversion of proton to neutron by emitting positron with the other pared particle called neutrino. This type of β-decay are commonly occur in the where one proton is required for the stability of the nucleus.

The equation (3) shows the electron capature by the parent nucleus. The proton in converting to the neutron by capturing the electron.
β^+-decay and ~~ek~~ electron capaturing will be occur only in case when more number of proton is required for the stability.

CALCULATION OF Q-VALUES

Considering the equation (1) we can calculate the Q-value:

$$Q = \left[m_N\left({}^{A}_{Z}X\right) - m_N\left({}^{A}_{Z+1}Y\right) - m_e - m_{\bar{\nu}} \right] c^2 \qquad (4)$$

Here $m_N\left({}^{A}_{Z}X\right)$ - nuclear mass of parent nucleus

$m_N\left({}^{A}_{Z+1}Y\right)$ - nuclear mass of daughter nucleus

m_e - mass of electron

$m_{\bar{\nu}}$ - mass of anti-neutrino.

By converting the nuclear mass to atomic mass equation (4) becomes

$$Q = \left[\left[m\left({}^{A}_{Z}X\right) - Zm_e \right] - \left[m\left({}^{A}_{Z+1}Y\right) - (Z+1)\, m_e \right] - m_e \right] c^2$$

Here $m\left({}^{A}_{Z}X\right)$ - atomic mass of parent

Zm_e - mass of electron for parent

$m\left({}^{A}_{Z+1}Y\right)$ - atomic mass of daughter

$(Z+1)\, m_e$ - mass of electron for daughter

$$\Rightarrow Q = \left[m\left({}^{A}_{Z}X\right) - m\left({}^{A}_{Z+1}Y\right) \right] c^2 \qquad (5)$$

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CALCULATION OF Q-VALUES

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$m_N\left({}^A_{Z+1} Y\right)$ - nuclear mass of daughter nucleus

m_e - mass of electron

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$$Q = \left[\left[m\left({}^A_Z X\right) - Z m_e \right] - \left[m\left({}^A_{Z+1} Y\right) - (Z+1) m_e \right] - m_e \right] c^2$$

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$Z m_e$ - mass of electrom for parent

$m\left({}^A_{Z+1} Y\right)$ - atomic mass of daughter

$(Z+1) m_e$ - mass of electron for daughter

$$\Rightarrow Q = \left[m\left({}^A_Z X\right) - m\left({}^A_{Z+1} Y\right) \right] c^2 \qquad (5)$$

Thus to calculate the Q-value for the β^--decay we have to look only to the atomic mass of the parent atom and daughter atom.

From the equation (2) for Q-value of β^+-decay

$$Q = \left[m_N\left({}^A_Z X\right) - m_N\left({}^{A}_{Z-1}Y\right) - m_e + m_\nu\right] c^2$$

$$= \left[\left[m\left({}^A_Z X\right) - Z m_e\right] - \left[m\left({}^{A}_{Z-1}Y\right) - (Z-1) m_e\right] - m_e\right]$$

$$\Rightarrow Q = \left[m\left({}^A_Z X\right) - m\left({}^{A}_{Z-1}Y\right) - 2 m_e\right] c^2 \qquad (6)$$

From the equation (3) for electron capturing

$$Q = \left[m_N\left({}^A_Z X\right) + m_e - m_N\left({}^{A}_{Z-1}Y\right)\right] c^2$$

$$= \left[m\left({}^A_Z X\right) - Z m_e + m_e - \left[m\left({}^{A}_{Z-1}Y\right) - (Z-1) m_e\right]\right] c^2$$

$$= \left[m\left({}^A_Z X\right) - m\left({}^{A}_{Z-1}Y\right)\right] c^2 \qquad (7)$$

*Note: Here we have neglected the mass of neutrino

· If we compare the β^+-decay and electron capturing (in both the case proton is converting to the neutron) we found that the Q-value of electron capturing is twice larger than the β^+-decay. Thus in many cases we have observed that electron capturing is energetically possible (but β^+-decay has Q-value negative in those cases).

Then the most puzzle of β-decay for the two decades is about the energy distribution of the β-decay.

If we consider some of the β-decay and its calculated Q-value. It shows that the Q-value is extra energy in the reduction in the rest mass energy which is appeared as the Kenetic energy of the product of daughter nuclei and parent nucleus β- particle it self and neutrino. Thus the energy of the β-decay ~~ev~~ is distributed among the daughter, electron and anti-neutrino. Here presently we have neglected the energy of the daughter nuclei. This is because the recoil energy of the daughter is very small as compared to the electron and neutrino (due to heavy mass of daughter nuclei). Thus the energy is shared between the electron and anti-neutrino. In the earlier experiment sceentist only measured the Kinetic energy of the electron (because neutrino and anti-neutrino are und-~~etect~~ ectable in those days).

Thus the scientist only consider the Q-value entirely by the electron (considering the rest mass of daughter and parent nuclei). Now let us consider the β-decay of ^{64}Cu to ^{64}Zn and ^{64}Ni

β^- decay

$$^{64}Cu \longrightarrow {}^{64}Zn + e^- + \bar{\nu}$$

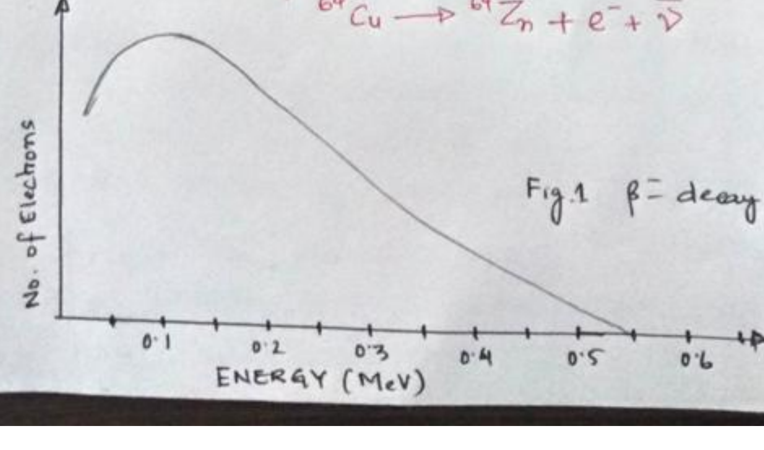


Fig.1 β^- decay

β^{+}- decay

$^{64}Cu \longrightarrow {}^{64}Ni + e^{+} + \nu$

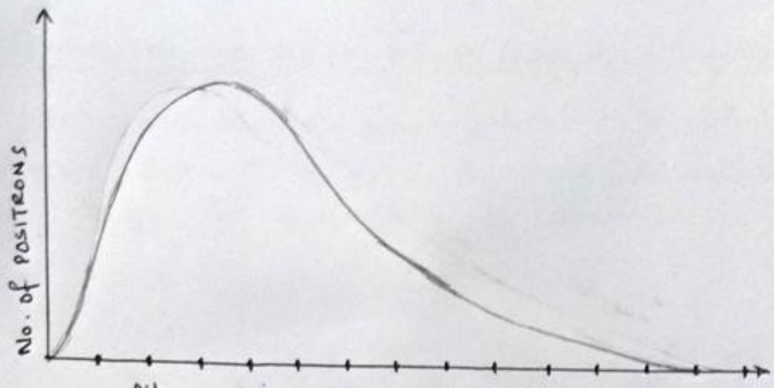


Fig. 2. β^{+}- decay

In both the fig 1 and 2 we observed the continuous spectra of β-decay. From these two plot if we calculate the kinetic energy of β^{+} and β^{-}- decay we have to consider only the rest mass of parent and daughter nuclei (which is the Q-value). Thus such type of approach will give the non mono energentic types of spectrae. or if consider the excited state the energy spectrum will be in discret spectrum.

From the experimental data it is found that the energy of the incoming electron is very much less than the Q-value in both β^{-}- decay and β^{+}- decay. Thus there is missing of some part of the energy.

In order to fill the missing energy Pauli proposed the hypothetical particle having neutral, lighter particle emitted along with the electron/positron which share the energy. Later this particle was named as neutrino by Fermi.

FERMI THEORY OF BETA DECAY (NON RELATIVISTIC)

Let us consider the wave functions ψ_i and ψ_f, which transforms from ψ_i to ψ_f. By time dependent perturbation theory the probability of transition is given by

$$\lambda = \frac{2\pi}{\hbar} \left| H_{if} \right|^2 \frac{dn}{dE_f} \qquad (8)$$

Equation (8) is called the Fermi- Golden Rule.

Here
H - interaction Hamiltonian

$\frac{dn}{dE_f}$ - density of state at final energy E_f.

Now let $\psi_i = \psi_p$, wave function of parent nuclei

$\psi_f = \psi_d \psi_e \psi_\nu$, wave function of daughter, electron and neutrino.

if we write the matrix element

$$H_{if} = \int \psi_d^* \psi_e^* \psi_\nu^* H \psi_p \, d\tau \qquad (9)$$

Both ψ_e^* and ψ_ν^* can be treated as free particle up to some extend.

We have

$$\psi_e = \frac{1}{\sqrt{V}} e^{i\vec{P}_e \cdot \vec{r}/\hbar} \qquad (9A)$$

$$\psi_\nu = \frac{1}{\sqrt{V}} e^{i\vec{P}_\nu \cdot \vec{r}/\hbar} \qquad (9B)$$

Here V - Volume of nucleus

As $\frac{\vec{P_e}\cdot\vec{r}}{\hbar}$ is small it kinetic energy in relativistic form is given by

$$\sqrt{P_e^2C^2 + m_0^2C^4} - m_0C^2 = KE. \qquad (10)$$

Let suppose $KE = 1\ MeV$, $m_0C^2 = 0.5\ MeV$

Putting value in (10)

$$P_eC = \sqrt{2}\ MeV$$

$$P_e = \frac{1.4}{c}\ MeV$$

$$\frac{\vec{P_e}\cdot\vec{r}}{\hbar} = \frac{\frac{1.4}{c}\ MeV \times 1\ fm}{\hbar}$$

$$\simeq 7\times10^{-3}$$

Thus it value is very small, $\frac{\vec{P_e}\cdot\vec{r}}{\hbar}$ can be expanded as

$$\psi_e = \frac{1}{\sqrt{V}}\left[1 + \frac{\vec{P_e}\cdot\vec{r}}{\hbar} + \cdots\right] \qquad (10A)$$

$$\psi_\nu = \frac{1}{\sqrt{V}}\left[1 + \frac{\vec{P_\nu}\cdot\vec{r}}{\hbar} + \cdots\right] \qquad (10B)$$

Putting the values in (9)

$$H_{if} = \frac{1}{V}\int \psi_d^* H \psi_p\, d\tau \qquad (11)$$

$$H_{if} = \frac{1}{V} M_{if} \qquad (11A)$$

M_{if} is called nuclear matrix element

In order to calculate the density of state $\frac{dn}{dE_f}$, let us consider a particle in cubical box of side L and which is free to move (no potential). The energy eigen states are given by

$$\psi_{n_1,n_2,n_3} = C_0 \sin \frac{n_1 \pi x}{L} \sin \frac{n_2 \pi y}{L} \sin \frac{n_3 \pi z}{L} \quad (12)$$

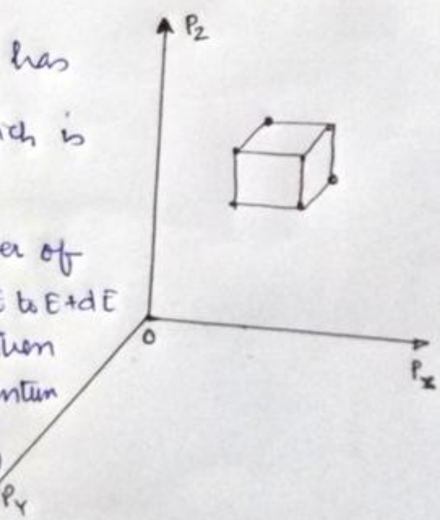
Here n_1, n_2 and n_3 describe the quantum states

$n = 1, 2, 3, \ldots\ldots$

The corresponding momentum is given by

$$P_x = \frac{n_1 \pi \hbar}{L}, \quad P_y = \frac{n_2 \pi \hbar}{L}, \quad P_z = \frac{n_3 \pi \hbar}{L}$$

If we plot all the quantum states in three dimensional momentum space it will form the cubical grid.



Each of the cubical grid has the volume of $\left(\frac{\pi\hbar}{L}\right)^3$ which is one quantum state.

~~If~~ To calculate the number of energy states from energy E to E+dE and momentum P to P+dP. Then the volume in the momentum space (P-space) is given by

$$V = \frac{1}{8}\left(4\pi p^2 dp\right) \quad (13)$$

Here $4\pi p^2 dp$ is the volume of the spherical shell of radius P. Thus there exist a shell form by P and P+dP

From the above case number of quantum states can be predicted correspond to one quantum state. Therefore total number of points on in cubical grite is

$$dn = \left(\frac{1}{8} 4\pi p^2 dp\right) / \left(\frac{\pi\hbar}{L}\right)^3 \qquad (14)$$

$$= \frac{1}{8} \frac{4\pi p^2 dp V}{(\pi\hbar)^3}$$

$$= \frac{4\pi p^2 dp V}{\hbar^3} \qquad (14A)$$

Thus

$$dn = \frac{4\pi P_e^2 dP_e V}{\hbar^3} \cdot \frac{4\pi P_\nu^2 dP_\nu V}{\hbar^3}$$

$$= (4\pi)^2 \frac{P_e^2 P_\nu^2 dP_e dP_\nu V^2}{(\hbar^3)^2} \qquad (14B)$$

The total energy of the β-decay is give by

$$E_f = m_e c^2 + K.E. + E_\nu$$

$m_e c^2$ — energy of electron

E_ν — " of neutrino

K.E. — Kenetic energy

$$E_f = m_e c^2 + K + P_\nu c \qquad (15)$$

Differentiating (15)

$$dE_f = dP_\nu c$$

$$\Rightarrow \frac{dP_\nu}{dE_f} = \frac{1}{c}$$

$$\frac{dn}{dE_f} = \frac{(4\pi)^2 P_e^2 P_\nu^2 dP_e}{\hbar^3} \times \frac{1}{c} \qquad (16)$$

Number of particle coming with kinetic energy K

$$N(K)\,dK = C_0\, P_e^2\, P_\nu^2\, dP_e \qquad (17)$$

The kinetic energy can have wide range of variation from very small to large value of Q, when the neutrino takes away the energy. Thus Q-value of electron energy

$$E^2 = P_e^2c^2 + m_e^2c^4 \qquad \text{(for electron)}$$

$$= (K + \text{rest mass energy})$$

$$= (K + m_ec^2)^2$$

$$= K^2 + 2Km_ec^2 + m_e^2c^2$$

$$\Rightarrow \quad P_e^2c^2 = K(K+2m_ec^2)$$

$$\Rightarrow \quad P_e^2 = \frac{K}{c^2}(K+2m_ec^2) \qquad (18A)$$

$$\Rightarrow \quad P_e = \frac{1}{c}\sqrt{K+2m_ec^2} \qquad (18B)$$

Again

$$Q = K + P_\nu c \qquad \text{(for neutrino)}$$

$$P_\nu = \frac{Q-K}{c}$$

$$\Rightarrow P_\nu^2 = \left(\frac{Q-K}{c}\right)^2 \qquad (18C)$$

Differentiating (18B) with respect to dK

$$dP_e = \frac{1}{c}\frac{2K+2m_eC^2}{\sqrt{(K+2m_eC^2)K}} \qquad (18D)$$

Putting all the values of P_ν, P_e and dP_e in (17)

$$N(K)\,dK = C_1\, K(K+2m_eC^2)(Q-K)^2\,\frac{(K+m_eC^2)}{\sqrt{K(K+2m_eC^2)}}\,dK$$

$$N(K) = C_1\, K(K+2m_eC^2)(Q-K)^2\,\frac{(K+m_eC^2)}{\sqrt{K(K+2m_eC^2)}}$$

$$N(K) = C_1\sqrt{K(K+2m_e c^2)}\ (Q-K)^2 (K+m_e c^2) \qquad (19)$$

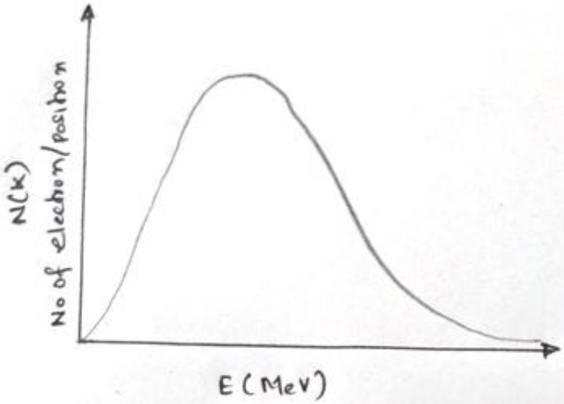


Fig. 3 Graph between N(K) and E

From the graph between the N(K) and E of the Fermi's finding doesnot match with the experimental data. A modification is needed in the assumption .i.e it need to consider the coloumbic intereaction between the daughter and emitted electron. Thus there is a modification in the wave function ~~of~~ of electron

$$\psi_e = \frac{1}{\sqrt{V}}\, e^{\frac{i\vec{P_e}\cdot\vec{r}}{\hbar}}$$

$$F(Z_d, K) = \frac{2\pi\eta}{1-e^{-2\pi\eta}} \qquad (20A)$$

$$\eta = \pm \frac{Z_d e^2}{4\pi\epsilon_0 \hbar v} \qquad (20B)$$

Here Z_d is no. of proton in daughter nuclei

v - velocity at which β- particle is emitted.

+ve → positron and -ve → electron

——— || ———

Then (19) became

$$N(K) = C_1 \sqrt{K(K+2m_e c^2)}\,(Q-K)^2 (K+m_e c^2)\, F(Z_d, K) \quad (21)$$

Equation (21) shows perfect matching between the theory and the experimental data.

REFERENCE

1. NUCLEAR PHYSICS – KAPLAN
2. NUCLEAR PHYSICS – EVANS