MOMENT OF INERTIA

The ***moment of inertia***, also known as the ***mass moment of inertia, angular mass*** *or* ***rotational inertia***, of a rigid body is a quantity that determines the torque needed for a desired angular acceleration about a rotational axis; similar to how mass determines the force needed for a desired acceleration. It depends on the body's mass distribution and the axis chosen, with larger moments requiring more torque to change the body's rotation rate.

It is an extensive (additive) property: for a point mass the moment of inertia is simply the mass times the square of the perpendicular distance to the rotation axis. The moment of inertia of a rigid composite system is the sum of the moments of inertia of its component subsystems (all taken about the same axis). Its simplest definition is the second moment of mass with respect to distance from an axis.

For bodies constrained to rotate in a plane, only their moment of inertia about an axis perpendicular to the plane, a scalar value, and matters. For bodies free to rotate in three dimensions, their moments can be described by a symmetric 3 × 3 matrix, with a set of mutually perpendicular principal axes for which this matrix is diagonal and torques around the axes act independently of each other.

When a body is free to rotate around an axis, torque must be applied to change its angular momentum. The amount of torque needed to cause any given angular acceleration (the rate of change in angular velocity) is proportional to the moment of inertia of the body. Moment of inertia may be expressed in units of kilogram meter squared (***kg·m^2***) in SI units and pound-foot-second squared (lbf·ft·s^2) in imperial or US units.

Moment of inertia plays the role in rotational kinetics that mass (inertia) plays in linear kinetics—both characterize the resistance of a body to changes in its motion. The moment of inertia depends on how mass is distributed around an axis of rotation, and will vary depending on the chosen axis. For a point-like mass, the moment of inertia about some axis is given by mr^2, where r is the distance of the point from the axis, and m is the mass. For an extended rigid body, the moment of inertia is just the sum of all the small pieces of mass multiplied by the square of their distances from the axis in rotation. For an extended body of a regular shape and uniform density, this summation sometimes produces a simple expression that depends on the dimensions, shape and total mass of the object.

Moment of inertia also appears in momentum, kinetic energy, and in Newton's laws of motion for a rigid body as a physical parameter that combines its shape and mass. There is an interesting difference in the way moment of inertia appears in planar and spatial movement. Planar movement has a single scalar that defines the moment of inertia, while for spatial movement the same calculations yield a 3 × 3 matrix of moments of inertia, called the inertia matrix or inertia tensor.

The moment of inertia of a rotating flywheel is used in a machine to resist variations in applied torque to smooth its rotational output. The moment of inertia of an airplane about its longitudinal, horizontal and vertical axes determine how steering forces on the control surfaces of its wings, elevators and rudder(s) affect the plane's motions in roll.

Moment of inertia I is defined as the ratio of the net angular momentum L of a system to its angular velocity ω around a principal axis, that is

$$I = \frac{L}{\omega}$$

If the angular momentum of a system is constant, then as the moment of inertia gets smaller, the angular velocity must increase. This occurs when spinning figure skaters pull in their outstretched arms or divers curl their bodies into a tuck position during a dive, to spin faster.

If the shape of the body does not change, then its moment of inertia appears in Newton's law of motion as the ratio of an applied torque $\boldsymbol{\tau}$ on a body to the angular acceleration $\boldsymbol{\alpha}$ around a principal axis, that is

$$\boldsymbol{\tau = I\alpha}$$

For a simple pendulum, this definition yields a formula for the moment of inertia $\boldsymbol{I}$ in terms of the mass $\boldsymbol{m}$ of the pendulum and its distance $\boldsymbol{r}$ from the axis point as,

$$\boldsymbol{I = mr^2}$$

Thus, moment of inertia of the pendulum depends on both the mass $\boldsymbol{m}$ of a body and its geometry, or shape, as defined by the distance $\boldsymbol{r}$ to the axis of rotation.

In general, given an object of mass $\boldsymbol{m}$, an effective radius $\boldsymbol{k}$ can be defined, dependent on a particular axis of rotation, with such a value that its moment of inertia around the axis is

$$\boldsymbol{I = mk^2}$$

where, $\boldsymbol{k}$ is known as the radius of gyration around the axis.