SCHOOL OF STUDIES IN PHYSICS JIWAJI UNIVERSITY GWALIOR

Topic: **Liquid Drop Model and Semi-emprical Mass Formula**

Nucleus

The **nucleus** is the center of an atom. It is made up of nucleons called (protons and neutrons) and is surrounded by the electron cloud. The size (diameter) of the nucleus is between 1.6 fm (10^{-15} m) (for a proton in light hydrogen) to about 15 fm (for the heaviest atoms, such as uranium). These sizes are much smaller than the size of the atom itself by a factor of about 23,000 (uranium) to about 145,000 (hydrogen). Although it is only a very small part of the atom, the nucleus has most of the mass. Almost all of the mass in an atom is made up from the protons and neutrons in the nucleus with a very small contribution from the orbiting electrons.

Neutrons have no charge and protons are positively charged. Because the nucleus is only made up of protons and neutrons it is positively charged. Things that have the same charge repel each other: this repulsion is part of what is called electromagnetic force. Unless there was something else holding the nucleus together it could not exist because the protons would push away from each other. The nucleus is actually held together by another force known as the strong nuclear force.

Nuclear Model

In order to explain how the nucleus looks like and how nucleons are held inside nucleus, several models have been proposed that help us to understand various nucleus features. The various models proposed are

- **Liquid Drop Model**
- **Nucleus Shell Model proposed independently**
- **Collective Model**

The *liquid-drop model*, which treats some of the gross collective features of nuclei in a way similar to the calculation of the properties of a droplet of liquid and the *shell model* deals more with individual nucleons. A successful model should satisfy two criteria

- It must reasonably explain the previously measured nuclear properties.
- It must predict additional properties that can be measured in new experiments.

Liquid Drop Model

The liquid drop model proposed by **George Gamow** and then developed by **N. Bohr** and **J.A. Wheeler**. Assumptions for the model are;

- According to this model, the atomic nucleus behaves **like the molecules in a drop** of liquid. But in this nuclear scale, the fluid is made of nucleons (protons and neutrons), which are held together by **the strong nuclear force**. Nuclear density is independent of nuclear volume as that of liquid drop.
- Nuclear forces are independent of spin and charge.
- The liquid drop model of the nucleus takes into account the fact that the nuclear forces on the nucleons on the surface are different from those on nucleons in the interior of the nucleus. The **interior nucleons are completely surrounded** by other attracting nucleons. Here is the analogy with the forces that form a drop of liquid.

- Emission of the radioactive rays is similar to release of water vapor when liquid drop is heated
- In the ground state the nucleus is **spherical**. If the sufficient kinetic or binding energy is added, this spherical nucleus may be distorted into a **dumbbell shape** and then may be splitted into **two fragments**. Since these fragments are a more stable configuration, the splitting of such heavy nuclei must be accompanied by **energy release**. This model does not explain all the properties of the atomic nucleus, but does explain the predicted nuclear binding energies.

Semi-emprical Mass Formula

One of the first models which could describe very well the behavior of the nuclear binding energies and therefore of nuclear masses was the mass formula of **von Weizsaecker** (also called **the semi-empirical mass formula – SEMF**), that was published in 1935 by German physicist **Carl Friedrich von Weizsäcker**.

The nuclear binding energy as a fuction of mass number A and the number of protrons Z based on the liquid drop model can be written as:

$$E_b(MeV) = a_V A - a_S A^{\frac{2}{3}} - a_C \frac{Z^2}{A^{\frac{1}{3}}} - a_A \frac{(A-2Z)^2}{A} \pm \delta(A,Z)$$

$$\delta(A,Z) = \begin{array}{c} +\delta_0 \; for\; Z,N\; even \\ 0 \\ -\delta_0 \; for\; Z,N\; odd \end{array}$$

This formula is called the Weizsaecker Formula (semi-emprical mass formula). The physical meaning of the equation can be discussed term by term.

Volume term – a_V.A. The first two terms describe a spherical liquid drop of an incompressible fluid with a contribution from the volume scaling with A and from the surface, scaling with $A^{2/3}$. The first positive term **a_V.A** is known as the volume term and it is caused by the attracting strong forces between the nucleons. The **strong force** has a **very limited range** and a given nucleon may only interact with its **direct neighbours**. Therefore this term is proportional to A, instead of A^2. The coefficient **a_V** is usually about ~ 16 MeV.

Surface term – a_{sf}.$A^{2/3}$. The surface term is also based on the strong force, it is, in fact, a **correction to the volume term**. The point is that particles at the surface of the nucleus are not completely surrounded by other particles. In the volume term, it is suggested that each nucleon interacts with a constant number of nucleons, independent of A. This assumption is very nearly true for nucleons deep within the nucleus, but **causes an overestimation** of the binding energy on the surface. By analogy with a liquid drop this effect is indicated as **the surface tension effect**. If the volume of the nucleus is proportional to A, then the geometrical radius should be proportional to $A^{1/3}$ and therefore the surface term must be proportional to the surface area i.e. proportional to $A^{2/3}$.

Coulomb term – $a_C.Z^2.A^{-⅓}$. This term describes the Coulomb repulsion between the uniformly distributed protons and is proportional to the number of proton pairs **Z^2/R**, whereby R is proportional to $A^{1/3}$. **This effect lowers the binding energy** because of the repulsion between charges of equal sign.

Asymmetry term – $a_A.(A-2Z)^2/A$. This term cannot be described as ‘classically’ as the first three. This effect is not based on any of the fundamental forces, this effect is based only on **the Pauli exclusion principle** (no two fermions can occupy exactly the same quantum state in an atom). The heavier nuclei contain more neutrons than protons. These extra neutrons are necessary for stability of the heavier nuclei. They provide (via the attractive forces between the neutrons and protons) some compensation for the repulsion between the protons. On the other hand, if there are significantly more neutrons than protons in a nucleus, some of the neutrons will be higher in energy level in the nucleus. This is the basis for a correction factor, the so-called symmetry term.

Pairing term – δ(A,Z). The last term is the pairing term δ(A,Z). This term captures the effect of spin-coupling. Nuclei with an even number of protons and an even number of neutrons are (due to Pauli exclusion principle) very stable thanks to the occurrence of ‘paired spin’. On the other hand, nuclei with an odd number of protons and neutrons are mostly unstable.

With the aid of **the Weizsaecker formula** the binding energy can be calculated very well for nearly all isotopes. This formula provides a good fit for heavier nuclei. For light nuclei, especially for ^{4}He, it provides a poor fit. The main reason is the formula does not consider the internal shell structure of the nucleus.

In order to calculate the binding energy, the coefficients a_V, a_S, a_C, a_A and a_P must be known. The coefficients have units of **megaelectron volts (MeV)** and are calculated **by** fitting **to** experimentally measured masses of nuclei**.** The coefficients in the equation are following:

$$E_b(MeV) = 15.76A - 17.81A^{2/3} - 0.711\frac{Z^2}{A^{1/3}} - 23.7\frac{(N-Z)^2}{A} \pm 34A^{-3/4}$$

Using the Weizsaecker formula, also the mass of an atomic nucleus can be derived and is given by:

$M = Z.m_p + N.m_n - E_b/c^2$

where **m_p** and **m_n** are the rest mass of a proton and a neutron, respectively, and **E_b** is the nuclear binding energy of the nucleus. From the nuclear binding energy curve it can be seen that, in the case of splitting a ^{235}U nucleus into two parts, the binding energy of the fragments (A ≈ 120) together is larger than that of the original ^{235}U nucleus. According to the Weizsaecker formula, the total energy released for such reaction will be approximately **235 x (8.5 – 7.6) ≈ 200 MeV.**