Topic: Isobaric Mass Parabola

The binding energy formula explains some of the important features of stability of nuclei, in particular the β -activity and stability properties of isobars.
From semi-empirical mass formula can have

$$M(Z, A) = Zm_p + Am_n - Zm_n - \{a_V A - a_S A^{\frac{2}{3}} - a_C \frac{Z^2}{A^{\frac{1}{3}}} - a_A \frac{(A-2Z)^2}{A} \pm \delta(A,Z)\} \quad (1)$$

This equation can be written as;

$$M(Z, A) = \alpha A + \beta Z + \gamma Z^2 \pm \delta(A, Z) \quad (2)$$

Where, $\alpha = m_n - a_V + a_A + a_S / A^{1/3}$; $\beta = -4a_A - (m_n - m_p)$ and $\gamma = \frac{4aA}{A} + \frac{a_C}{A^{1/3}}$

Equation (2) is quadratic in 'Z'. Therefore, if we plot a graph between M(Z, A) *vs* Z , it would be a parabola like curve

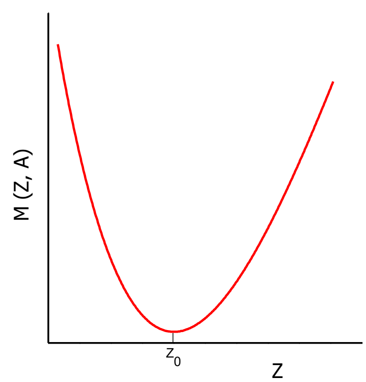


Fig. 1: M(Z, A) *vs* Z parabolic curve

The bottom of the curve represents the most stable nucleus of series, with highest binding energy. All the isobars with binding energy less than the most stable one (at bottom) will lie at the arms of the curve. They will decay by β -emission of electron (e^-) or positron (e^+) or by K-capture. At Z = Z_0, we have minimum of curve.

The condition for minimum is $\left.\frac{\partial M}{\partial Z}\right|_{A=constant} = 0$

From equation (2), we have $\frac{\partial M}{\partial Z} = \beta + 2\gamma Z$

At Z = Z_0 ; $\frac{\partial M}{\partial Z} = 0$

So, $Z_0 = -\beta / 2\gamma$ or $\beta = -Z_0 2\gamma \quad (3)$

Therefore at Z_0; $M(Z, A) = \alpha A - \gamma Z_0^{\ 2} - \{\pm\delta(A, Z)\} \quad (4)$

Depending upon the number of proton and neutrons inside the nucleus we can have different nuclear configuration as given in Table 1.

Table 1: Different configurations of nuclei

Z	**N**	**A**	**Pairing energy** $\delta(A,Z)$
Odd	Even	Odd	0
Even	Odd	Odd	0
Odd	Odd	Even	- δ_0
Even	Even	Even	δ_0

During a β-decay either electron is released (β^-) or a positron is emitted (β^+) from the nucleus. The process total nucleons (A) remains the same, however proton is conversed into neutron and vice-versa, thereby Z and N changes. The reactions for the processes can be written as;

$$ {}^{A}_{Z}X_{N} \xrightarrow{\beta^{+}decay} {}^{A}_{Z-1}Y_{N+1} + \beta^{+} + \nu + E_{\beta^{+}} \quad (5)$$

$$ {}^{A}_{Z}X_{N} \xrightarrow{\beta^{-}decay} {}^{A}_{Z+1}Y_{N\_1} + \beta^{-} + \bar{\nu} + E_{\beta^{-}} \quad (6)$$

ν and $\bar{\nu}$ are neutrino and antineutrino released during decay. E_{β^+} and E_{β^-} is the energy released and should be positive. Therefore, M(X) should be greater than M(Y).

Odd A Isobars

Odd A isobars have either odd Z or odd N but not both odd simultaneously. The pairing energy vanishes for such alloys i.e., $\delta(A,Z)=0$. Therefore, from equation (2), we have

$$M(Z,A) = \alpha A + \beta Z + \gamma Z^2$$

By substituting value of β from equation (3), we get

$$M(Z,A) = \alpha A - 2\gamma Z_0 Z + \gamma Z^2$$

$$M(Z,A) = \gamma (Z - Z_0)^2 + \alpha A - \gamma Z_0^2$$

On Changing Z to Z+1 or Z to Z-1, we get

$$M(Z+1,A) = \gamma (Z+1-Z_0)^2 + \alpha A - \gamma Z_0^2$$

$$M(Z-1,A) = \gamma (Z-1-Z_0)^2 + \alpha A - \gamma Z_0^2 \quad (7)$$

During β^+ decay energy released is;

$$E_{\beta^+} = M(Z,A) - M(Z-1,A)$$

Therefore using equation (7) we have

$$E_{\beta^+} = 2\gamma (Z - Z_0 - 1/2)$$

For β^- decay energy released given by

$$E_{\beta^-} = M(Z,A) - M(Z+1,A)$$

$$E_{\beta^-} = 2\gamma (Z_0 - Z - 1/2)$$

If binding energy for a series of nuclei with constant odd A and varying Z is plotted against Z. The result is a parabola like curve, shown in the Figure 2 (A). In case of odd A series, pairing energy is zero, therefore we get only one parabola.

The isobar at the bottom is the most stable one. The isobars on the left of the most stable have fewer protons compared to it, decay by electron emission.

$$Te \xrightarrow{\beta-} I \xrightarrow{\beta-} Xe \xrightarrow{\beta-} Cs \xrightarrow{\beta-} Ba$$

Isobars on the right side of stable isobar have excess protons decay by **positron** emission or by K-capture or by both.

$$Ce \xrightarrow{\beta+} La \xrightarrow{\beta+} Ba$$

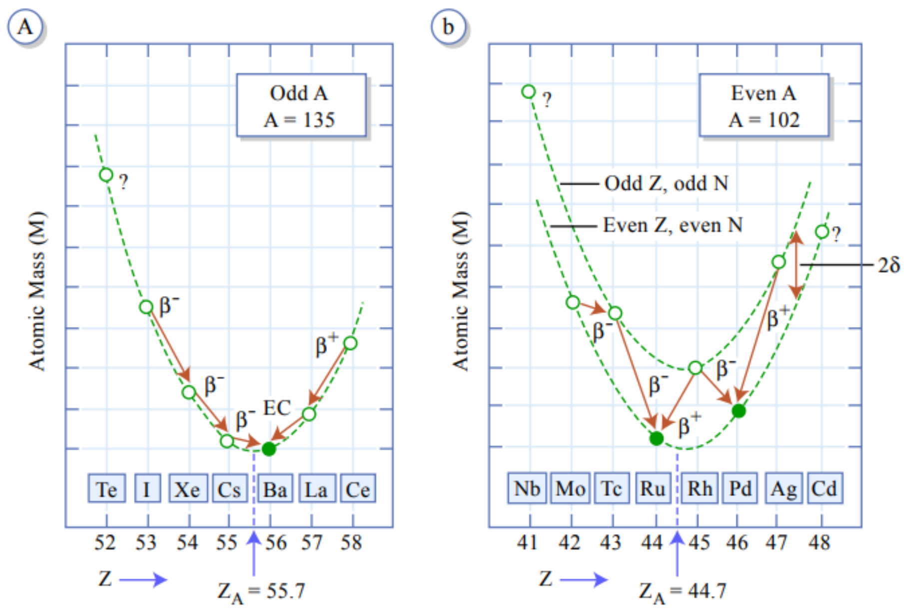


Fig. 2: Isobaric mass parabola (A) odd A nuclei; (B) even A nuclei.

Even A Isobars

The result obtained for even A nuclei is different from odd A nuclei because of odd-even effect. In even A-nuclei pairing energy is not zero. Since both odd-odd and even-even nuclei have even A, we therefore have two different values of pairing energy given in table above. Therefore, we have two parabolas in binding energy curve displaced by $2\delta_0$.

For even-even nuclei

$$M(Z,A) = \gamma(Z - Z_0)^2 + \alpha A - \gamma Z_0^{\ 2} - \delta_0 \qquad (8)$$

For odd-odd nuclei

$$M(Z,A) = \gamma(Z - Z_0)^2 + \alpha A - \gamma Z_0^{\ 2} + \delta_0 \qquad (9)$$

By changing Z to Z-1 and Z to Z+1 one can obtain relations for M (Z-1, A) and M (Z+1, A). From these equations (8 and 9) it is clear that even-even nuclei are more stable than odd–odd nuclei. For β^- decay of odd-odd nuclei

$$E_{\beta^-} = M(Z,A) - M[(Z+1),A]$$

odd-odd   even-even

Using equations (8) and (9), we have

$$E_{\beta^-} = 2\gamma[(Z_0 - Z) - 1/2] + 2\delta_0$$

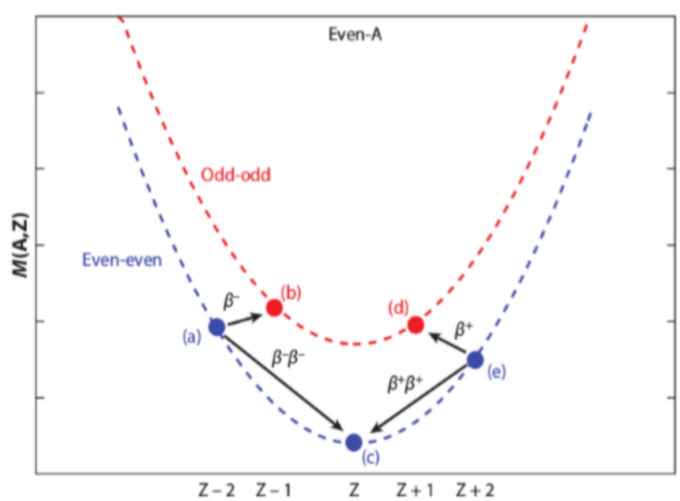
For β $^+$ decay of odd-odd nuclei

$$E_{\beta^-} = 2\gamma[-(Z_0 - Z) - 1/2] - 2\delta_0$$

The isobaric mass parabola for even A nuclei is shown in Fig. 2 (B). The odd-odd nuclei lie on the upper curve is unstable with respect to even-even nuclei; thereby undergo β -decay to attain stability. Nuclear processes in which two protons simultaneously to become two neutrons or vice-versa is known as double β -decay. This is the reason why for even-even nuclei have two or more stable nuclei. The A = 136 isobaric family has three stable nuclei.

Comment on Stability and the Mass Parabola

By minimizing the semi-empirical mass formula as a function of the proton number, we found that it is possible for there to be multiple stable nuclei for a given *A* for even-*A* nuclei.



The Valley of Stability