

# Comparison between Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac statistics

These three statistics concern when how particles occupy a system which consists of several energy levels (and each energy level could also have several energy states). A particle in this system can be in one of those energy levels depending on the energy particle has. It's impossible to have just one particle in a system since in real life it needs various particles to constitute a system. They occupy the levels under a statistics rule. There are three statistics:

- Particles which are regulated by *Maxwell-Boltzmann Statistics* have to be distinguishable each other and one energy state can be occupied by two or more particles. Distinguishable means that if we have 2 particles, let say A and B, also two states, 1 and 2, and we put A to state 1 and B to state 2, it will be different with the distribution A to state 2 and B to state 1. It means that A and B are distinct.
- Particles which are regulated by *Bose-Einstein Statistics* have to be indistinguishable each other and one energy state can be occupied by two or more particles. So instead of saying it as particle A or B, we call it as just “particle” since they are the same thing.

- Particles which are regulated by *Fermi-Dirac Statistics* have to be indistinguishable each other and one energy state can be occupied by only one particle. So we have to fill it to another state when a state has just been occupied by another particle.
- **Maxwell Boltzmann** statistics is applicable to identical, *distinguishable* particle of any type spin. The molecules of gas are particle of this type.
- **Bose Einstein** statistics is applicable to the identical, *indistinguishable* particles of zero or integral spin. These particles are called Bosons. Example photons, Helium atom.
- **Fermi Dirac** statistics is applicable to the identical, indistinguishable particles of half integral spin. These particles obey **Pauli Exclusion Principle** .Ex Electron, proton etc.
- *Maxwell–Boltzmann statistics* apply to “classical” particles with non-quantized energy levels. It turns out there aren’t any truly classical particles, but M-B statistics is still a good approximation in many cases.
- Bose–Einstein statistics apply to quantum particles with the property that any number of particles can occupy any level. It turns out these are the particles with integral “spin” such as photons and Helium-4 (but not Helium-3) atoms.

- Fermi–Dirac statistics - apply to quantum particles with the property that only one particle can occupy any particular level. It turns out these are the particles with half-integral “spin” such as electrons, neutrons and Helium-3 atoms.
- Both BE and FD statistics converge (from opposite directions) on M-B statistics when the density of particles is small compared to the Quantum concentration, where the inter-particle distance is comparable to the de Broglie wavelength.

1. Maxwell-Boltzmann

$$f(E) = A e^{-E/kT}$$

→ Distinguishable  
 → unlimited number of particles per state  
 examples: (gas molecules)

2. Bose-Einstein

$$f(E) = \frac{1}{\left[ \exp\left(\frac{E}{kT}\right) - 1 \right]}$$

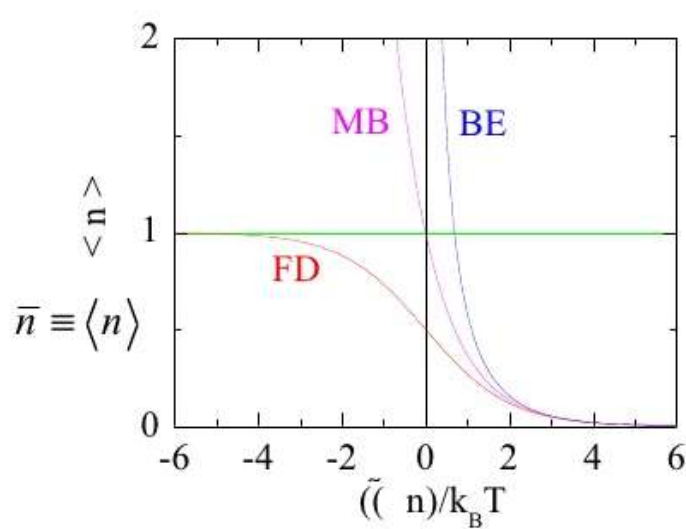
→ Indistinguishable  
 → unlimited number of particles per state  
 (photon)

3. Fermi-Dirac

$$f(E) = \frac{1}{\left[ \exp\left(\frac{E-E_f}{kT}\right) + 1 \right]}$$

→ Indistinguishable  
 → only one particle per state  
 $E_f$  = fermion energy  
 (electron)

# Comparison of FD, BE and MB Distribution



$$\bar{n}_{FD}(\epsilon) = \frac{1}{\exp\left(\frac{\epsilon - \mu}{k_B T}\right) + 1}$$

$$\bar{n}_{BE}(\epsilon) = \frac{1}{\exp\left(\frac{\epsilon - \mu}{k_B T}\right) - 1}$$

$$\bar{n}_{MB}(\epsilon) = \exp\left(-\frac{\epsilon - \mu}{k_B T}\right)$$

what are the possible values of  $\mu_{MB}$ ,  $\mu_{FD}$ , and  $\mu_{BE}$  ?

$$\mu_{MB} < 0 \quad \mu_{FD} < \epsilon_F (> 0) \quad \mu_{BE} < \min(\epsilon) \approx 0$$

Boltzmann	Bose Einstein	Fermi Dirac
$\bar{n}_k = \frac{1}{\exp\left(\frac{\epsilon - \mu}{k_B T}\right)}$	$\bar{n}_k = \frac{1}{\exp\left(\frac{\epsilon - \mu}{k_B T}\right) - 1}$	$\bar{n}_k = \frac{1}{\exp\left(\frac{\epsilon - \mu}{k_B T}\right) + 1}$
indistinguishable $Z = (Z_1)^N / N!$ $n_k \ll 1$	indistinguishable integer spin 0, 1, 2 ...	indistinguishable half-integer spin 1/2, 3/2, 5/2 ...
spin doesn't matter	bosons	fermions
localized particles $\Psi$ don't overlap	wavefunctions overlap total $\Psi$ symmetric	wavefunctions overlap total $\Psi$ anti-symmetric
gas molecules at low densities	photons $^4\text{He}$ atoms	free electrons in metals electrons in white dwarfs
"unlimited" number of particles per state $n \ll 1$	unlimited number of particles per state	never more than 1 particle per state

## • Equilibrium Distributions for Different Statistics

Goal : Find the occupation in each energy level when the thermodynamics probability is maximum (equilibrium point).

Model	Particle Properties	Case
<b>Maxwell-Boltzmann</b>	distinguishable unlimited particles per quantum state	ideal gas molecules
<b>Bose-Einstein</b>	indistinguishable unlimited particles per quantum state	photon, phonon
<b>Fermi-Dirac</b>	indistinguishable, identical one particle per quantum state (Pauli exclusion principle)	electron, protons

- MB can be considered as the limiting case of BE or FD model.