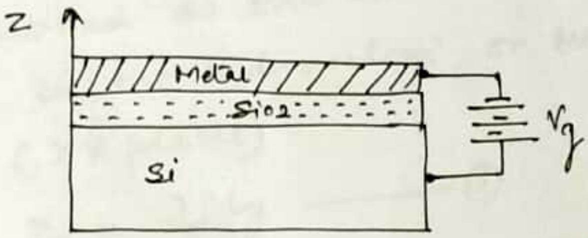
Integral quantum Hall effect

The Hall effect based on classical considerations, gives a good account of the electrical transport in metals and semiconductors. But this classical magneto-conducting scenario undergoes a spectacular transformation under quantum conditions of temperature and magnetic field in a two dimensional conductivity channel.

K. Von. Klitzing, Dorda and Pepper observed that such a channel is formed at the oxide interface in a metal-oxide-semiconductor (M.O.S) transistor when a gate voltage is applied between the metal and the semiconductor.



The fascinating aspect of their observation is that the Hall resistance ρ_H varies with the magnetic field according to the following rule;

$$\rho_H = \frac{h}{i e^2}$$ where 'i' is an integer ($= 1, 2, 3 \ldots$)

or $\sigma_H = i \frac{e^2}{h}$

The phenomenon expressed by this rule, where the Hall conductance is quantised in units of e^2/h, is called the Integral quantum Hall effect (IQHE)

In the original experiment a constant current of 1 μA was forced to flow between the source and drain in the presence of a magnetic field of 18 tesla at 1.5 kelvin.

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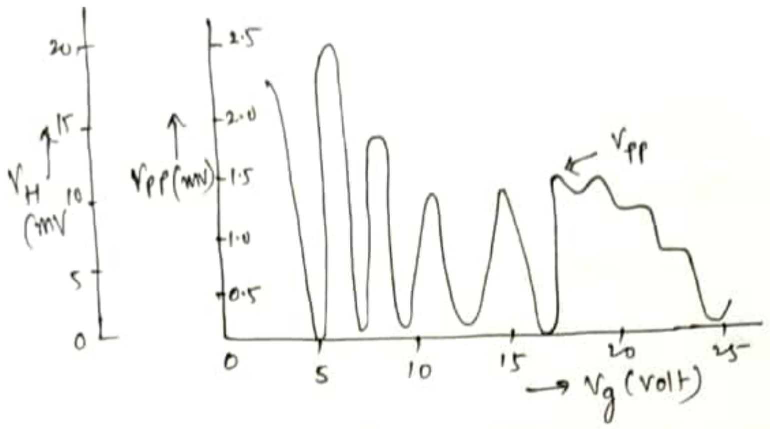

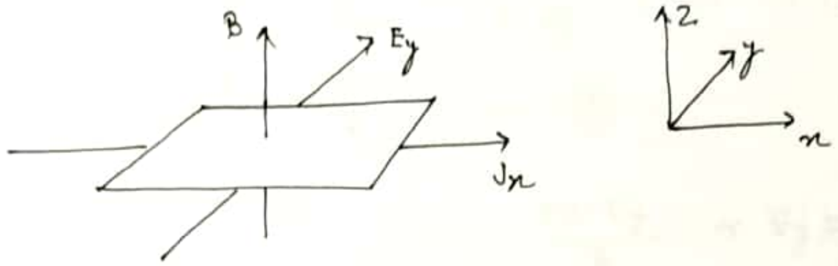

Consider a surface current density $\vec{J_x}$ in the x-direction defined as the current crossing a line of unit length in the y. direction on the oxide interface in (x y plane)

$$I_x = J_x L_y \quad —— (1)$$

If 'n' is the total electron density per unit area of the x-y plane

$$j_x = n e v_d \quad —— (2)$$

$v_d \to$ drift velocity of electron in x-direction.

In the description of Hall effect, a current in a specimen with mobile charge carriers is produced when the specimen is placed in a region of mutually crossed electric and magnetic fields. The flow of current in a direction orthogonal to both the fields is detected on closing the circuit. If electric field $\vec{E_y}$ and magnetic field $\vec{B}$ act along y- and z-directions, respectively

$$j_x = \sigma_{xx} E_x + \sigma_{xy} E_y = \sigma_{xy} E_y \quad —— (3)$$

(when $E_x = 0$)

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and drift velocity

$$V_d = \frac{E_y}{B} \quad \text{——} (4)$$

Here σ_{xy} denotes conductivity Tensor in the plane of two dimensional chhannel. writting the resistivity ρ_{xy} as Hall resistance ρ_H

$$\rho_H = \frac{V_H}{I_x} = \frac{E_y L_y}{J_x L_y} \quad [\text{using eq}(1)] \left[\begin{array}{l}\text{as } V_H \text{ developed along } y\text{-direction so} \\ \frac{V_H}{L_y} = E_y \text{ or} \\ V_H = E_y L_y\end{array}\right]$$

$$\text{or } \rho_H = \frac{E_y}{j_x} \quad \text{——} (5)$$

form (2) and (4) $\quad j_x = \frac{ne \cdot E_y}{B}$ or $E_y = \frac{B \cdot j_x}{ne}$ — (6)

using (5) and (6)

$$\rho_H = \frac{B \cdot j_x}{ne} \frac{1}{j_x} = \frac{B}{ne} \quad \text{—} (7)$$

This represent the resistance of a chhannel of unit thickness.

In a magnetic field $\vec{H}$, an electrons experiences a Lorentz force. ⊥ to the field.

$$m \frac{d^2 \vec{r}}{dt^2} = e\left(\frac{\partial \vec{r}}{\partial t}\right) \times \vec{B} \quad \text{——} (i)$$

In x-y plane the inward directed Magnetic force $e(\vec{u} \times \vec{H})$ give rise to motion in a circle of radius $\vec{r}$ with a cyclotron freequency. 'ω'

$$\frac{mu^2}{r} = e\,uB \quad \text{or} \quad \frac{u}{r} = \frac{eB}{m}$$

$$\text{and} \quad \omega = \frac{2\pi}{T} = 2\pi \Big/ \frac{2\pi r}{u} = \frac{u}{r}$$

$$\Rightarrow \quad \omega = \frac{Be}{m} \quad \text{or} \quad \frac{u}{r} = \frac{Be}{m} \quad \text{or} \quad B = \frac{mu}{er}$$

as m, e, u are constant we have.

$$B \propto 1/r \quad \text{——} (ii)$$

$\Rightarrow$ when $\vec{B}$ is large, we have very small orbit radii,

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According to correspondence principle the energy associated to these small radii orbit is quantized. (4)

$$E_\perp = (n+\tfrac{1}{2})\hbar w, \quad \text{where } n \text{ is an integer.}$$

In a thin film of thickness d_z, the component of wave vector k_z ($\perp$ar to the surface) also get quantized in units of $\frac{2\pi}{d_z}$

Thus we have

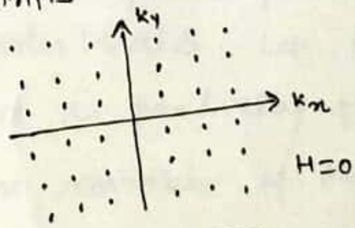
$$E_\perp = (n+\tfrac{1}{2})\hbar w_c \qquad n = 0,1,2,3 \ldots (x\text{-}y \text{ plane})$$

$$\text{and } (k_z)_l = \left(\frac{2\pi}{d_z}\right)_l \qquad l = 0, \pm 1, \pm 2 \ldots (\text{along } z \text{ axis})$$

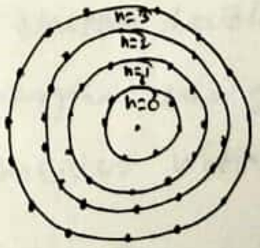
So we have $E_{nl} = E_\perp + E_z = (n+\frac{1}{2})\hbar w + \left(\frac{\hbar^2 k_z^2}{2m}\right)_l$.

At low temperature $\underline{T \to 0}$, for small d_z, the levels are separated enough to stratify the electron states. These quantized state (E_{nl}) are called Landau levels.

In the absence of magnetic field the allowed electron states in two dimension are represented by points in k-space. $\frac{\hbar^2}{2m}(k_x^2 + k_y^2)$ and shown by



When sufficient strong magnetic field is applied these points corresponding to states of free electrons may be viewed as restricted to concentric circles in $k_x k_y$ plane. The successive circles corresponds to successive values of the quantum number '$\underline{n}$' in the energy $(n+\frac{1}{2})\hbar w_c$. The area in between the successive circles are



$$\left[\begin{array}{l} E = \frac{\hbar^2 k^2}{2m} \text{ or } k^2 = \frac{2mE}{\hbar^2} \\ \text{or } 2k\Delta k = \frac{2m}{\hbar^2}\Delta E \\ k\Delta k = \frac{m}{\hbar^2}\Delta E \end{array}\right.$$

$$\pi\Delta(\Delta k)^2 = 2\pi k(\Delta k) = 2\pi\frac{m}{\hbar^2}\Delta E$$

$$= \frac{2\pi m}{\hbar^2}\hbar w_c = \frac{2\pi m w_c}{\hbar} = \frac{2\pi m}{\hbar}\cdot\frac{Be}{m} = \frac{2\pi Be}{\hbar}$$

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Thus the area in between successive circles are proportional to B as $\frac{2\pi e}{\hbar}$ are constant.

The number of states on each circle is constant and is equal to the area between the circles times the number of states per unit area when B=0. The number of states per unit area when B=0 are $\left(\frac{L}{2\pi}\right)^2$

Thus the no. of states on each circle is

$$\frac{2\pi Be}{\hbar} \times \left(\frac{L}{2\pi}\right)^2 = \frac{\cancel{2}\pi Be}{\hbar}\frac{L^2}{\cancel{4}\pi^2} = \frac{Be}{2\hbar\pi}L^2 = \frac{BeL^2}{2\pi \cdot h/2\pi} = \frac{BeL^2}{h}$$

It gives the number of electron states or levels. that coalesce into a single magnetic level as soon as even a small magnetic field is switched on. Thus in fact this describes the degeneracy of a Landau level. Therefore degeneracy per unit area in x-y plane is.

$$D(B) = \frac{BeL^2}{h} \Big/ L^2 = \frac{Be}{h} \quad \text{——} \quad (\text{IV})$$

When the applied magnetic field is so strong that $\hbar w_c \gg k_B T$. we can talk in terms of completely filled or ~~[illegible]~~ completely empty Landau levels. Let B_i is critical field at which no level is partially filled and 'i' is the magnetic quantum number of the highest occupied level. When the electron density on the oxide interface is adjusted by varying the gate voltage so that the fermilevel coincides with the level 'i'

$$n = \cancel{i(B)}\; i\, D(B_i) \quad \text{——} \quad (\text{V})$$

$$= i\,\frac{B_i e}{h}$$

from equation (6) $J_n = \frac{n.e\,E_y}{B}$ where 'n' being the density of charge carriers. —— (VI)

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$$R_H = \frac{E_y}{J_x B_z} = \frac{J_x}{\sigma_{xy}} \cdot \frac{1}{J_x B_z} = \frac{\rho_{xy}}{B_z} \quad \text{using eq. } 3$$

Thus $$\sigma_{xy} = \frac{j_x}{E_y} = \frac{ne}{B_i} \overset{\text{using } 4}{=} \frac{i B_i e}{h} \cdot \frac{e}{B_i} \quad \text{using } V$$

or $$\sigma_{xy} = i\frac{e^2}{h} \quad \text{——— (VI)}$$

Thus the Hall conductivity of the two dimensional system in the quantum limit (smaller orbit) is quantised in integral multiples of e^2/h. These narrow levels are lying between the Fermi level E_F.

Under the condition the electron can undergo either elastic or inelastic scattering. The elastic scattering would lead to an electron to go in the same Landau level. But it is not permitted by pauli exclusion principle, since all possible final states of equal energy are occupied. The inelastic collision can be possible with the scattering of electron to a high level by absorbing the energy from some source. The possible source may be phonon, but at low temperature and with $\hbar\omega_c \gg k_BT$, there are hardly any phonon whose energy could compare with the large energy interval $\hbar\omega_c$. Thus inelastic ~~energy~~ collisions are too ruled out and electron mean path is greatly enhanced. This results in the occurrance of the voltage minima in V_{pp}.

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Quantum hall effect demonstration

A two dymensional electron system exists in a metal oxide semiconductor field effect-transistor i.e. MOSFET. In an n-channel MOSFET the fermi level E_F can be made to enter the conduction band by applying a large positive gate voltage V_g. This create an inversion layer which forms a two dimensional electron gas. These inversion layers are formed at the interface between a semiconductor and an insulator or in between two semiconductors, with one of them acting as insulator. The system in which the quantum Hall effect was discovered has Si for semiconductors and SiO_2 for the insulator. Recently the semiconductor - semiconductor system GaAs - $Al_x Ga_{(1-x)} As$, the later playing the Role of the insulator, has also used to study this effect

The principle of inversion layer is quite simple. It is arranged that an electric field perpendicular to the interface attracts electrons from the semiconductor to it. These electrons are sit in the quantum well created by this field and the interface. The motion perpendicular to the interface is quantized and thus the motional degree of freedom in this direction is ~~restricted~~. freezes out. The result is a net two dimensional system of electron.

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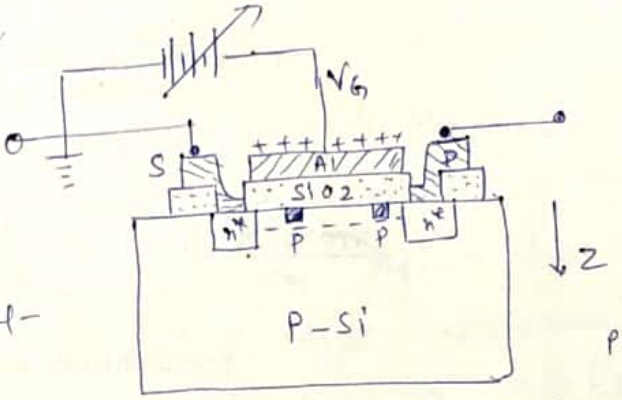
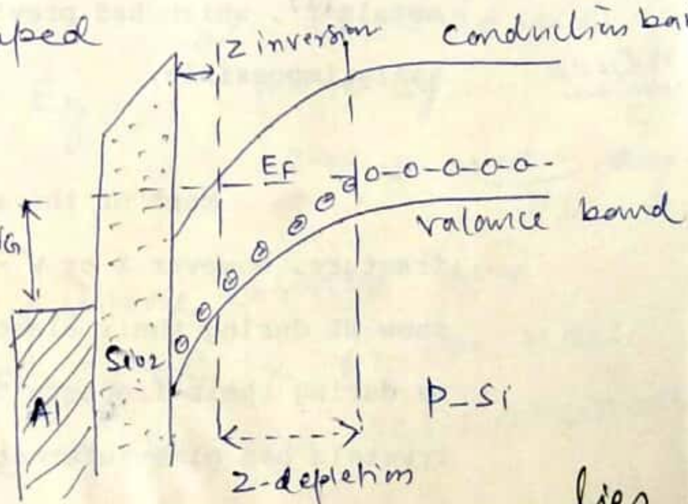
The source of attracting electric field differs in the two system. In the Silicon case the insulating oxide layer is made relatively thick (5000 Å) and a metallic electrode (Al) plated over it. This electrode is positively charged by applying the positively charged external gate voltage. This arrangement of the electric field permit providing a better control over the density of sheet of electrons attracted to the interface.



The later self consistent potential seen by the picture of electrons can be conveniently by the periodic band bending. That is to say, the periodic lattice potential gives rise to energy bands and slowly varying electric potential then is regarded as bending of these bands.

The semiconductor is doped with p type i.e. some electrons of the valence band have become bound to acceptor dopant impurities leaving empty states, holes. The holes are lowest energy at the top of the valence band. This means that the fermi level lies close to the top of that band. The electrons first fill up these hole attracted to the surface



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states leaving a net negative charge of three dimensional density equal to the acceptor density. However if there is sufficient gate voltage, the bottom of the conduction band will become lower than the fermi level, and if there is a way for them to get there the electrons will occupy states in the part of the conduction band bends below the fermi level near the insulator-semiconductor interface. This layer is called inversion layer, with the bottom of the conduction band below the top of the valence band. inverting the normal order. It is also possible to use a n-type semiconductor and reverse the sign of the gate voltage to achieve inverted hole states.

Let a current j_x flow along x-direction and a magnetic field H be applied along the z-direction then as $j_x = -n e \langle u_x \rangle$ and $E_y = -\frac{\langle u_x \rangle H_z}{c}$ then $j_x = +\frac{n_e c}{H_z} E_y$

$$j_x = \frac{n_e ec}{H} E_y$$

or $\quad j_x = \sigma_{xy} E_y$

where n_e is the density of charge carriers and $\sigma_{xy} = \left(\frac{n_e ec}{H}\right)$, which relate E_y and j_x is the Hall conductivity. The reciprocal of this is Hall resistivity ρ_{xy} and.

$$R_H = \frac{E_y}{j_x H_z} = \frac{j_x}{\sigma_{xy}} \cdot \frac{1}{j_x H_z} = \frac{\rho_{xy}}{H_z}$$ is the Hall coefficient. with the carrier concentration

$n_e = n_L = g_L \nu \quad$ where $\nu = 1, 2, 3 \ldots$

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Thus. $\sigma_{xy} = \frac{j_x}{E_y} = \frac{n_e \cdot ec}{H} = \frac{g_L \nu\, ec}{H}$

But $g_L = \frac{e}{hc} H \Rightarrow n_e = g_L \nu = \frac{eH}{hc}\nu$

$\sigma_{xy} = \frac{eH\nu}{h\cancel{c}} \cdot \frac{e\cancel{c}}{\cancel{H}} = \nu\left(\frac{e^2}{h}\right)$ Thus $\sigma_{xy} = \frac{n_e ec}{H} = \nu\left(\frac{e^2}{h}\right)$ where $\nu = 1, 2, 3 \ldots$

Thus the Hall conductivity of the two dimensional system (inversion layer) in the quantum limit (small orbit) is quantized in integral multiples of e^2/h. These ν narrow levels are lying below E_F i.e. fermi level.

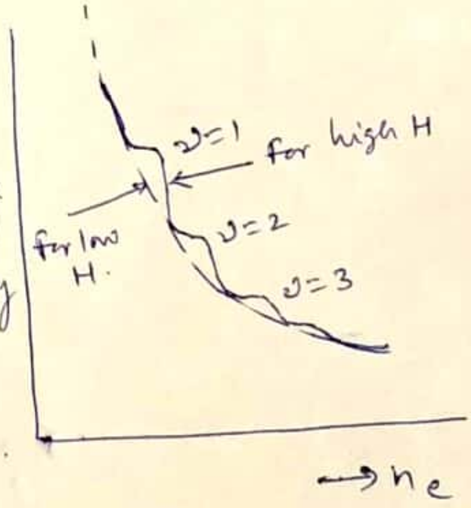
Klitzing received the 1985 Nobel prize for observing the quantum Hall effect by measuring the quantum Hall effect by measuring ρ_{xy} in silicon MOSFET and so e^2/h (or $e^2/\hbar c$) accurately. As $\rho_{xy} \propto 1/n_e$ the sign and value of n_e is found by measuring ρ_{xy}. $\left[\sigma_{xy} = \frac{n_e ec}{H} \text{ or } \rho_{xy} = \frac{1}{\sigma_{xy}} = \frac{H}{n_e ec} \text{ or } \rho_{xy} \propto \frac{1}{n_e}\right.$

When the potential difference is applied across the SiO_2 crystal in a MOSFET, an inversion layer (25-50 Å thick) is formed at the SiO_2-Si interface. The density of electrons in the layer depends on the potential difference. The inversion layer region is a rectangular box of thickness 'd_2'. The lowest excitation energy $\left[\frac{\hbar^2}{2m}(k_z^2)_l\right]$ for the motion of the electrons along z-axis is about 20 milli eV. Below 10K temperature, the typical energies are only about

1 milli ev. Thus the motion low to the layer (9)
is frozen out and it behaves as a two dimensional layer.

Klitzing et al varied V_g (that is n_e) with a constant source-drain current J_x and measured the Hall voltage, which gave ρ_{xy}. For small H, the expected smooth monotonic decrease of ρ_{xy} with n_e, $\rho_{xy} \propto 1/n_e$ was found.

The fig. shows the plot of ρ_{xy} vs n_e at small H. Dashed curve indicate $\rho_{xy} \propto 1/n_e$ and large H (Quantum Hall effect) showing steps according to $\left[\rho_{xy} = \frac{h}{\nu e^2}\right]$ $\nu = 1, 2, 3 \ldots$



To have $\hbar\omega > kT$ or $\frac{\hbar He}{mc} > kT$

or $\frac{\hbar e}{mc} H > kT$ or $1.6 \times 10^{-20} H > 1.38 \times 10^{-16} T$

for $T = 4.2k$ we get $H > 42$ kilogauss. When $T = 1.5k$ and $H = 180$ kilo gauss was kept, they found that over certain ranges of n_e (surface charge density) the ρ_{xy} remains constant (Hall steps). These steps are the manifestation of quantization and occur at $\rho_{xy} = h/\nu e^2$. [illegible]

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The value of $e^2/\hbar c$ is found like $1/137.0353$. It is also found that, unlike the ordinary Hall effect, the normal resistivity measured by potential probes vanishes, as in a superconductor, in the region of Hall steps. If E_f for some (V_g) is between two Landau levels, the density of states at E_f is zero and an inversion layer carrier cannot be scattered. Then the flow of current is lossless. The normal resistivity vanishes at Hall steps.

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Integral quantum Halls effect

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