ASSIGNMENT
ON
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Eigen Functions, Eigen Values and Stationary States

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Eigen Functions, Eigen Values and Stationary States

* **Eigen Functions :-** A state vector $|\psi\rangle$ is said to be an eigen vector of an operator $\hat{A}$. If the operator of $\hat{A}$ to $|\psi\rangle$ gives.

$$\hat{A}|\psi\rangle = a|\psi\rangle$$

where 'a' is a complex number called an eigen value of $\hat{A}$. This equation is known as eigen value equation of the operator $\hat{A}$.

Example :- A simple example is the eigen value problem for the unity operator $\hat{I}$

$$\hat{I}|\psi\rangle = |\psi\rangle$$

This means that all vectors are eigen vectors of $\hat{I}$ with one eigen value 1.

" Let an operator $\hat{A}$ operators upon a wave function ψ to produce a new function ψ' which is also a function of the same variable as that of ψ. In some cases we write

$$\hat{A}\psi = \psi' = \lambda\psi.$$

where λ is a constant, then the wave function ψ is said to be the eigen function of $\hat{A}$ and λ is called the eigen value of $\hat{A}$. Also the equation

$$\hat{A}\psi = \lambda\psi$$

is known as Eigen value equation.

⇒ **Properties of eigen function :-**

1) **Simultoneous eigen function :-** Eigen vector operator different eigen function.

$$\frac{d}{dx} e^{ikx} = ik e^{ikx}$$

$$\frac{d^2}{dx^2} e^{ikx} = -k^2 e^{ikx}$$

2) **Orthogonal and orthonormal :-** Let ψ_1 and ψ_2 be two eigen functions.

$$\int_a^b \psi_1^*(r,t) . \psi_1(r,t)\, d\tau = 1$$

$$\int_a^b \psi_2^*(r,t) . \psi_2(r,t)\, d\tau = 0$$

3) **Degeneracy :-** Two different eigen function having some eigen values.

$$\left.\begin{aligned} H_1\psi_1 &= E\psi_1 \\ H_2\psi_2 &= E\psi_2 \end{aligned}\right] \text{ deegeneracy.} \qquad \left.\begin{aligned} H_1\psi_1 &= E_1\psi_1 \\ H_2\psi_2 &= E_2\psi_2 \end{aligned}\right] \text{ non-degeneracy.}$$

⇒ **Stationary States :-** In a particular state if the probability distribution, $\psi^*\psi$ is independent of time, then the state of the system is said to be stationary states.

Let, us consider the probability distribution function $\psi^*\psi$ for a system in the state represented by the wave function

$$\psi(x,y,z,t) = \sum_{n=1}^{\infty} a_n \phi_n(x,y,z)\, e^{-iE_nt/\hbar} \longrightarrow (1)$$

Its conjugate is represented by

$$\psi^*(x,y,z,t) = \sum_{n=1}^{\infty} a_m^* \phi_m^*(x,y,z)\, e^{iE_mt/\hbar}$$

so that,

$$\psi^*\cdot\psi = \sum_{m=1}^{\infty} a_m^* \phi_m^*(x,y,z) e^{iE_mt/\hbar} \cdot \sum_{n=1} a_n \phi_n(x,y,z) e^{-iE_nt/\hbar}$$

$$\psi^*\psi = \sum_{n=1}^{\infty} (a_n a_m^* \phi_n(x,y,z) \cdot \phi_m^*(x,y,z)\, e^{iE_mt/\hbar} \cdot e^{-iE_nt/\hbar})$$

$$\psi^*\psi = \sum a_n a_m^* \phi_n(x,y,z)\, \phi_m^*(x,y,z) + \sum_m \sum_n{}' a_n a_m^* \phi_n(x,y,z)\, \phi_m^*(x,y,z) \cdot e^{i(E_m-E_n)t/\hbar}$$

where the prime on the double summation indicates that the terms with $m-n$ are executed.

The equation ① is not a stationary state solution $\psi^* \psi$ with be independent of time t. only if a_n are zero for all values except for one value of E_n.

In the case the wave function will certain only, a single term and will be represented by

$$\psi_n(x,y,z,t) = \phi_n(x,y,z)\, e^{-iE_n t/\hbar} \longrightarrow (2)$$

The function $\phi_n(x,y,z)$ being a particular solution of time independent Schrodinger equation.

The solution $\psi_n(x,y,z,t) = \phi_n(x,y,z)\, e^{-iE_n t/\hbar}$ is stationary state, since $\psi^* \psi = \phi_m^* \phi_n$ which is independent of time.