

Deepa Anwar

ANOVA for comparing means between more than 2 groups

Hypotheses of One-Way ANOVA

- $H_0: \mu_1 = \mu_2 = \mu_3 = \dots = \mu_c$
 - All population means are equal
 - i.e., no treatment effect (no variation in means among groups)
- \mathbf{H}_{1} : Not all of the population means are the same
 - At least one population mean is different
 - i.e., there is a treatment effect
 - Does not mean that all population means are different (some pairs may be the same)

The F-distribution

• A ratio of variances follows an F-distribution:

$$\frac{\sigma_{between}^2}{\sigma_{within}^2} \sim F_{n,m}$$

• The F-test tests the hypothesis that two variances are equal.

• F will be close to 1 if sample variances are equal.

$$\begin{aligned} H_0 : \sigma_{between}^2 &= \sigma_{within}^2 \\ H_a : \sigma_{between}^2 \neq \sigma_{within}^2 \end{aligned}$$

ANOVA Table

| Source of variation | d.f. | Sum of squares | Mean Sum of Squares | F-statistic | p-value |
|--|------|---|-------------------------------------|--|---|
| Between (k groups) | k-1 | SSB (sum of squared deviations of group means from grand mean) | | $\frac{\frac{SSB}{k-1}}{\frac{SSW}{nk-k}}$ | Go to F _{k-1,nk-k} chart |
| Within (n individuals per group) | nk-k | SSW (sum of squared deviations of observations from their group mean) | s ²⁼ SSW/nk-k | | |
| Total variation | nk-1 | ~~~~~ | ed deviations of rom grand mean) | TSS=SSB + SS | SW |

Example

| Treatment 1 | Treatment 2 | Treatment 3 | Treatment 4 |
|-------------|-------------|-------------|-------------|
| 60 inches | 50 | 48 | 47 |
| 67 | 52 | 49 | 67 |
| 42 | 43 | 50 | 54 |
| 67 | 67 | 55 | 67 |
| 56 | 67 | 56 | 68 |
| 62 | 59 | 61 | 65 |
| 64 | 67 | 61 | 65 |
| 59 | 64 | 60 | 56 |
| 72 | 63 | 59 | 60 |
| 71 | 65 | 64 | 65 |

Example

Step 1) calculate the sum of squares between groups:

Mean for group 1 = 62.0

Mean for group 2 = 59.7

Mean for group 3 = 56.3

Mean for group 4 = 61.4

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| 56 | 67 | 56 | 68 |
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| 64 | 67 | 61 | 65 |
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Grand mean = 59.85

 $SSB = [(62-59.85)^2 + (59.7-59.85)^2 + (56.3-59.85)^2 + (61.4-59.85)^2] xn per group = 19.65x10 = 196.5$

Example

Step 2) calculate the sum of squares within groups:

 $(60-62)^{2}+(67-62)^{2}+(42-62)^{2}+(67-62)^{2}+(62-62)^{2}+(64-62)^{2}+(59-62)^{2}+(72-62)^{2}+(71-62)^{2}+(50-59.7)^{2}+(52-59.7)^{2}+(43-59.7)^{2}+(52-59.7)^{2}+(67-59.7)^{2}+(67-59.7)^{2}+(69-59.7)^{2}-(67-59.7)^{2}+(69-59.7)^{2}-(67-59.7)^{2}+(69-59.7)^{2}-(67-59.7)^{2}+(69-59.7)^{2}-(67-59.7)^{2}+(69-59.7)^{2}-(67-59.7)^{2}+(69-59.7)^{2}-(67-59.7)^{2}+(69-59.7)^{2}-(67-59.7)^{2}+(69-59.7)^{2}-(67-59.7)^{2}+(69-59.7)^{2}-(67-59.7)^{2}+(69-59.7)^{2}-(67-59.7)^{2}+(69-59.7)^{2}-(67-59.7)^{2}+(69-59.7)^{2}-(67-$

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Step 3) Fill in the ANOVA table

| Source of variation | <u>d.f.</u> | Sum of squares | Mean Sum of <u>Squares</u> | <u>F-statistic</u> | <u>p-value</u> |
|---------------------|-------------|----------------|-------------------------------|--------------------|----------------|
| Between | 3 | 196.5 | 65.5 | 1.14 | .344 |
| Within | 36 | 2060.6 | 57.2 | _ | _ |
| Total | 39 | 2257.1 | _ | _ | _ |

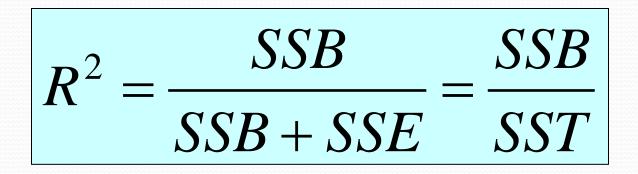
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INTERPRETATION of ANOVA:

How much of the variance in height is explained by treatment group? $R^{2="Coefficient of Determination" = SSB/TSS = 196.5/2275.1=9\%$

Coefficient of Determination



The amount of variation in the outcome variable (dependent variable) that is explained by the predictor (independent variable).

Beyond one-way ANOVA

Often, you may want to test more than 1 treatment. ANOVA can accommodate more than 1 treatment or factor, so long as they are independent. Again, the variation partitions beautifully!

TSS = SSB1 + SSB2 + SSW