Common terms of significance

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Hypothesis Testing with One Sample

Introduction to Hypothesis Testing

Hypothesis Tests

A **hypothesis test** is a process that uses sample statistics to test a claim about the value of a population parameter.

If a manufacturer of rechargeable batteries claims that the batteries they produce are good for an average of at least 1,000 charges, a sample would be taken to test this claim.

A verbal statement, or claim, about a population parameter is called a **statistical hypothesis**.

To test the average of 1000 hours, a pair of hypotheses are stated – one that represents the claim and the other, its complement. When one of these hypotheses is false, the other must be true.

Stating a Hypothesis

"H subzero" or "H naught"

A **null hypothesis** H_0 is a statistical hypothesis that contains a statement of equality such as $\leq$, $=$, or $\geq$.

"H sub-a"

A **alternative hypothesis** H_a is the complement of the null hypothesis. It is a statement that must be true if H_0 is false and contains a statement of inequality such as $>$, $\neq$, or $<$.

To write the null and alternative hypotheses, translate the claim made about the population parameter from a verbal statement to a mathematical statement.

Stating a Hypothesis

Example:
Write the claim as a mathematical sentence. State the null and alternative hypotheses and identify which represents the claim.

A manufacturer claims that its rechargeable batteries have an average life of at least 1,000 charges.

$\mu \geq 1000$

Condition of equality

H_0: $\mu \geq 1000$ (Claim)
H_a: $\mu < 1000$

Complement of the null hypothesis

Stating a Hypothesis

Example:

Write the claim as a mathematical sentence. State the null and alternative hypotheses and identify which represents the claim.

Statesville college claims that 94% of their graduates find employment within six months of graduation.

$p = 0.94$

→ Condition of equality

H_0: $p = 0.94$ (Claim)

H_a: $p \neq 0.94$

→ Complement of the null hypothesis

Types of Errors

No matter which hypothesis represents the claim, always begin the hypothesis test **assuming that the null hypothesis is true**.

At the end of the test, one of two decisions will be made:

1. reject the null hypothesis, or
2. fail to reject the null hypothesis.

A type I error occurs if the null hypothesis is rejected when it is true.

A type II error occurs if the null hypothesis is not rejected when it is false.

Types of Errors

	Actual Truth of H_0	
Decision	**H_0 is true**	**H_0 is false**
Do not reject H_0	Correct Decision	**Type II Error**
Reject H_0	**Type I Error**	Correct Decision

Types of Errors

Example:

Statesville college claims that 94% of their graduates find employment within six months of graduation. What will a type I or type II error be?

H_0: $p = 0.94$ (Claim)

H_a: $p \neq 0.94$

A type I error is rejecting the null when it is true.

The population proportion is actually 0.94, but is rejected. (We believe it is not 0.94.)

A type II error is failing to reject the null when it is false.

The population proportion is not 0.94, but is not rejected. (We believe it is 0.94.)

Level of Significance

In a hypothesis test, the level of significance is your maximum allowable probability of making a type I error. It is denoted by α, the lowercase Greek letter alpha.

Hypothesis tests are based on α.

The probability of making a type II error is denoted by β, the lowercase Greek letter beta.

By setting the level of significance at a small value, you are saying that you want the probability of rejecting a true null hypothesis to be small.

Commonly used levels of significance:

$\alpha = 0.10$ $\alpha = 0.05$ $\alpha = 0.01$

Statistical Tests

After stating the null and alternative hypotheses and specifying the level of significance, a random sample is taken from the population and sample statistics are calculated.

The statistic that is compared with the parameter in the null hypothesis is called the test statistic.

Population parameter	Test statistic	Standardized test statistic
μ	$\overline{x}$	$z \ (n \geq 30)$ $t \ (n < 30)$
p	$\hat{p}$	z
σ^2	s^2	X^2

P-values

If the null hypothesis is true, a ***P*-value** (or **probability value**) of a hypothesis test is the probability of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data.

The *P*-value of a hypothesis test depends on the nature of the test.

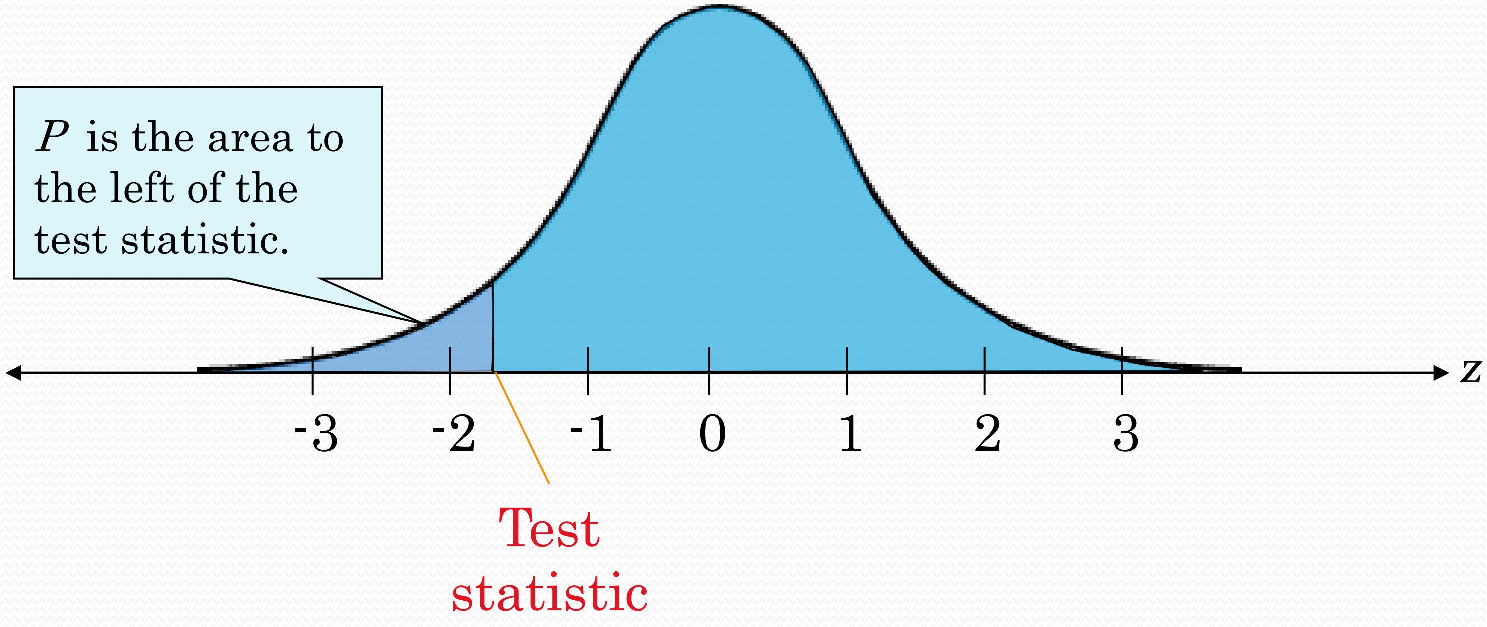
There are three types of hypothesis tests – a left-, right-, or two-tailed test. The type of test depends on the region of the sampling distribution that favors a rejection of H_0. This region is indicated by the alternative hypothesis.

Left-tailed Test

1. If the alternative hypothesis contains the less-than inequality symbol (<), the hypothesis test is a left-tailed test.

H_0: $\mu \geq k$

H_a: $\mu < k$

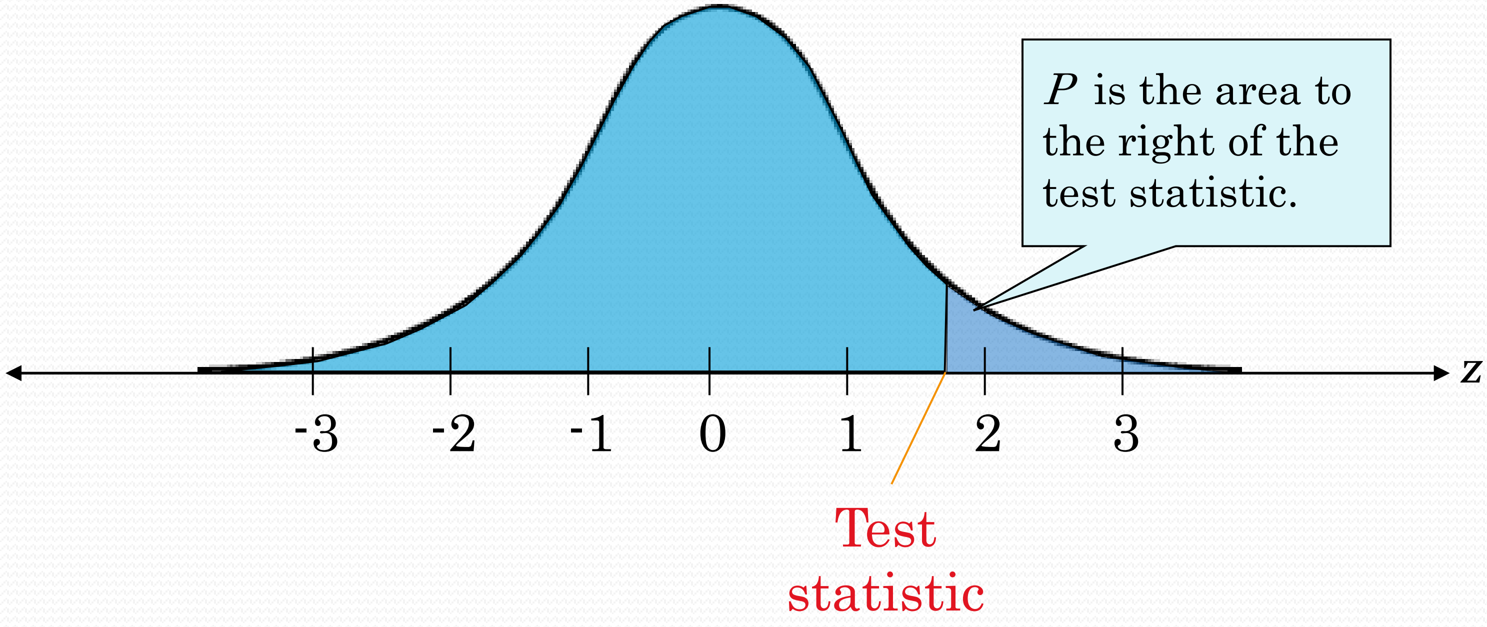


Right-tailed Test

2. If the alternative hypothesis contains the greater-than symbol (>), the hypothesis test is a right-tailed test.

H_0: $\mu \leq k$

H_a: $\mu > k$

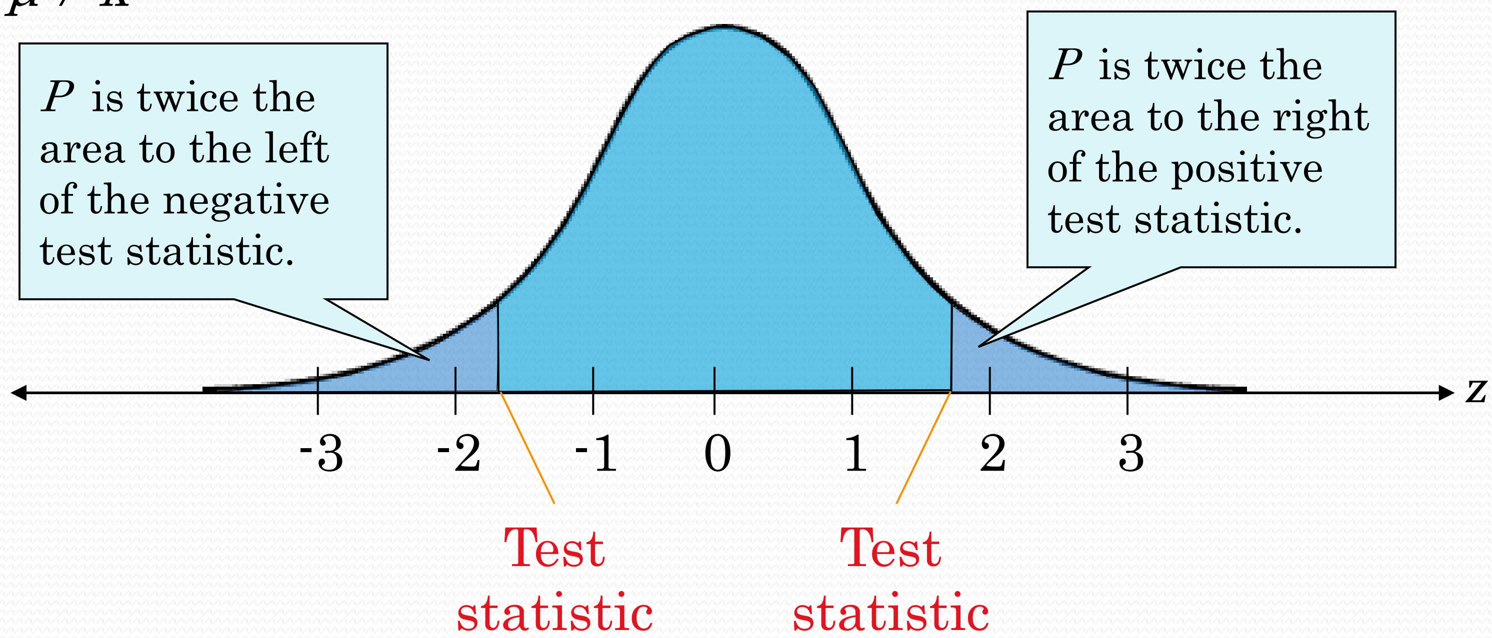


Two-tailed Test

3. If the alternative hypothesis contains the not-equal-to symbol (≠), the hypothesis test is a two-tailed test. In a two-tailed test, each tail has an area of $\frac{1}{2}P$.

H_0: $\mu = k$

H_a: $\mu \neq k$



Identifying Types of Tests

Example:

For each claim, state H_0 and H_a. Then determine whether the hypothesis test is a left-tailed, right-tailed, or two-tailed test.

a.) A cigarette manufacturer claims that less than one-eighth of the US adult population smokes cigarettes.

H_0: $p \geq 0.125$

H_a: $p < 0.125$ (Claim)

Left-tailed test

b.) A local telephone company claims that the average length of a phone call is 8 minutes.

H_0: $\mu = 8$ (Claim)

H_a: $\mu \neq 8$

Two-tailed test

Making a Decision

Decision Rule Based on *P*-value

To use a *P*-value to make a conclusion in a hypothesis test, compare the *P*-value with α.

1. If $P \leq \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

	Claim	
Decision	**Claim is H_0**	**Claim is H_a**
Reject H_0	There is enough evidence to reject the claim.	There is enough evidence to support the claim.
Do not reject H_0	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

Interpreting a Decision

Example:

You perform a hypothesis test for the following claim. How should you interpret your decision if you reject H_0? If you fail to reject H_0?

H_0: (Claim) A cigarette manufacturer claims that less than one-eighth of the US adult population smokes cigarettes.

If H_0 is rejected, you should conclude "there is sufficient evidence to indicate that the manufacturer's claim is false."

If you fail to reject H_0, you should conclude "there is *not* sufficient evidence to indicate that the manufacturer's claim is false."

Steps for Hypothesis Testing

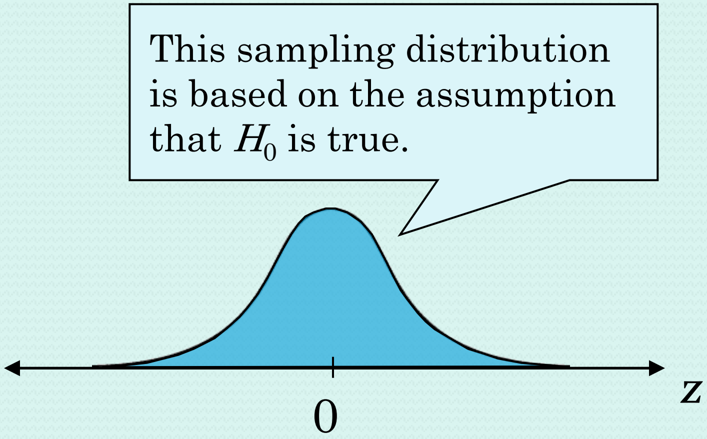
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.

   H_0: ? H_a: ?

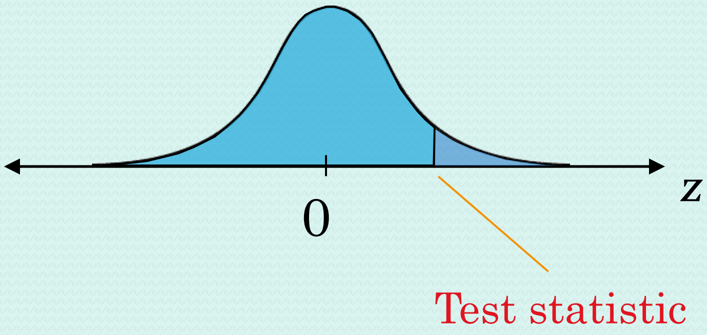
2. Specify the level of significance.

   $\alpha = ?$

3. Determine the standardized sampling distribution and draw its graph.



4. Calculate the test statistic and its standardized value. Add it to your sketch.



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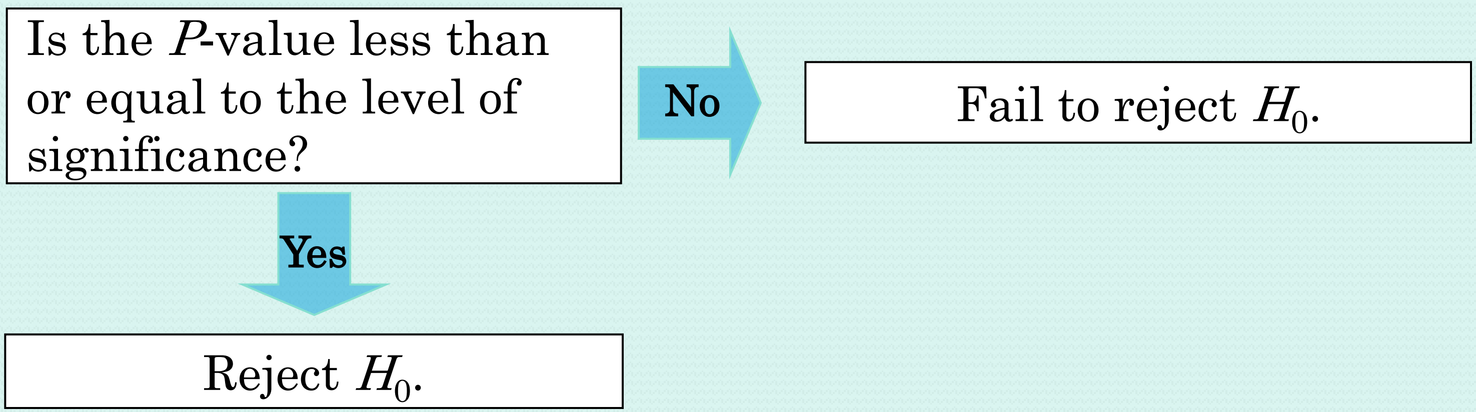
Steps for Hypothesis Testing

5. Find the *P*-value.
6. Use the following decision rule.



7. Write a statement to interpret the decision in the context of the original claim.

These steps apply to left-tailed, right-tailed, and two-tailed tests.