GEOMETRIC MEAN, WEIGHTED GEOMETRIC MEAN

Geometric mean

The geometric mean of a series containing n observations is the nth root of the product of the values.

If x1, x2…, xn are observations then

$$G.M = \sqrt[n]{x_1, x_2 \ldots x_n}$$

$$= (x_1, x_2 \ldots x_n)^{1/n}$$

$$\text{Log GM} = \frac{1}{n}\log(x_1, x_2 \ldots x_n)$$

$$= \frac{1}{n}(\log x_1 + \log x_2 + \ldots + \log x_n)$$

$$= \frac{\sum \log x_i}{n}$$

$$GM = \text{Antilog } \frac{\sum \log x_i}{n}$$

For grouped data

$$GM = \text{Antilog } \left[\frac{\sum f \log x_i}{n}\right]$$

GM is used in studies like bacterial growth, cell division, etc.

If the weights of sorghum ear heads are 45, 60, 48,100, 65 gms. Find the Geometric mean for the following data

Weight of ear head x (g)	**Log x**
45	1.653
60	1.778
48	1.681
100	2.000
65	1.813
Total	**8.925**

Solution

Here n = 5

$$GM = \text{Antilog} \frac{\sum \log x_i}{n}$$

$$= \text{Antilog} \frac{8.925}{5}$$

$$= \text{Antilog } 1.785$$

$$= 60.95$$

Grouped Data

Find the Geometric mean for the following

Weight of sorghum (x)	No. of ear head(f)
50	4
65	6
75	16
80	8
95	7
100	4

Weight of sorghum (x)	No. of ear head(f)	Log x	f x log x
50	5	1.699	8.495
63	10	10.799	17.99
65	5	1.813	9.065
130	15	2.114	31.71
135	15	2.130	31.95
Total	**50**	**9.555**	**99.21**

Here n= 50

$$GM = \text{Antilog}\left[\frac{\sum f \log x_i}{n}\right]$$

$$= \text{Antilog}\left[\frac{99.21}{50}\right]$$

$$= \text{Antilog } 1.9842 = 96.43$$

Solution will be like this

Continuous distribution

For the frequency distribution of weights of sorghum ear-heads given in table below. Calculate the Geometric mean

Weights of ear heads (in g)	No of ear heads (f)
60-80	22
80-100	38
100-120	45
120-140	35
140-160	20
Total	**160**

Solution

Weights of ear heads (in g)	No of ear heads (f)	Mid x	Log x	f log x
60-80	22	70	1.845	40 59
80-100	38	90	1.954	74.25
100-120	45	110	2.041	91.85
120-140	35	130	2.114	73.99
140-160	20	150	2.176	43.52
Total	**160**			**324.2**

Here $n = 160$

$$GM = \text{Antilog}\left[\frac{\sum f \log x_i}{n}\right]$$

$$= \text{Antilog}\left[\frac{324.2}{160}\right]$$

$$= \text{Antilog}\left[2.02625\right]$$

$$= 106.23$$

WEIGHTED GEOMETRIC MEAN

Let $x_1, x_2, \ldots x_k$ be the k values of the variable and $w_1, w_2, \ldots w_k$ be associated weights.Then W.G.M. or G_w, called the weighted geometric mean of $x_1, x_2, \ldots x_k$ is given by

$$W.G.M. \text{ or } G_w = \text{Antilog}\left[\frac{\Sigma w \log x}{\Sigma w}\right]$$

where, W.G.M. = represents Weighted Geometric Mean

$x_1, x_2, \ldots, x_n$ = represent the values of items

$w_1, w_2, \ldots, w_n$ = represent the weights corresponding to the size of items to which they relate

$\Sigma(w \times \log x)$ = represent the total of the product of weights and logs of the size of items

Σw = represent total of weights

Illustration 12

Calculate weighted geometric mean of the following data :

Size of Item	25	.003	5.72	0.46	.0008
Weight	5	8	4	7	6

Solution

x	w	log x	w log x
25	5	1.3979	6.9895
0.003	8	$\bar{3}$.4771	$\bar{21}$.8168
5.72	4	0.7574	3.0296
0.46	7	$\bar{1}$.6628	$\bar{1}$.6396
0.0008	6	$\bar{4}$.9031	$\bar{19}$.4186
			$\bar{32}$.8941
Total	30		or −31.1059

$$\text{Weighted Geometric Mean, } G_w = \text{Antilog}\left(\frac{\Sigma w \log x}{\Sigma w}\right)$$

$$= \text{Antilog}\left(\frac{-31.1059}{30}\right)$$

$$= \text{Antilog}(-1.0369)$$

$$= \text{Antilog}(\bar{2}.9631) = 0.09185$$

- https://www.youtube.com/watch?v=VjYv0snDBP4