

Institute of Engineering,
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Unit-IV BE IV Sem

WAVE PROPAGATION IN DIELECTRIC
EL-402 ELECTRONICS
EMT

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WAVE PROPAGATION IN DIELECTRIC →

Let uniform plane wave to propagation in dielectric of permittivity ϵ and permeability μ . The medium is isotropic and homogeneous, and the wave equation

$$\nabla^2 E_s = -k^2 E_s \quad \text{--- (1)}$$

where the wave number is now a function of the material properties:

$$k = \omega \sqrt{\mu \epsilon} = k_0 \sqrt{\mu_r \epsilon_r} \quad \text{--- (2)}$$

From E_{xs} we have

$$\frac{d^2 E_{xs}}{dz^2} = -k^2 E_{xs} \quad \text{--- (3)}$$

$$E_{xs} = E_{x0} e^{-jkz} = E_{x0} e^{-\alpha z} e^{-j\beta z} \quad \text{--- (4)}$$

Multiplying (4) by $e^{j\omega t}$

$$E_x = E_{x0} e^{-\alpha z} \cos(\omega t - \beta z) \quad \text{--- (I)}$$

$$jk = \alpha + j\beta$$

$k =$ complex propagation constant

Complex Permittivity →

The ways in which physical processes in a material can affect the wave electric field are described through a complex permittivity of the form.

$$\epsilon = \epsilon' - j\epsilon'' \quad (5)$$

Two important mechanisms that give rise to a complex permittivity are -

- i. Bound electron or ion oscillations
- ii. Dipole relaxation
- iii. Conduction of free electrons or holes

In terms of wave losses in magnetic field,

$$\mu = \mu' - j\mu'' \quad (6)$$

Note - The magnetic response is usually very weak compared to the dielectric response in most materials of interest for wave propagation; in such materials, $\mu \approx \mu_0$.

Loss tangent →

From ϵ_q^n (3) (4) & (5)

$$k = \omega \sqrt{\mu(\epsilon' - j\epsilon'')} \\ = \omega \sqrt{\mu\epsilon'} \sqrt{1 - j \frac{\epsilon''}{\epsilon'}} \quad - (7)$$

$$(\alpha + j\beta)^2 = \\ (\alpha^2 - \beta^2) + j2\alpha\beta \\ = -\omega^2 \mu \epsilon' + j\omega^2 \mu \epsilon''$$

$$\alpha^2 - \beta^2 = -\omega^2 \mu \epsilon' \\ 2\alpha\beta = \omega^2 \mu \epsilon''$$

$$\alpha + j\beta = j\omega \sqrt{\mu\epsilon'} \left(1 - j \frac{\epsilon''}{\omega\epsilon'}\right)^{1/2} \quad (i) \\ j\omega \sqrt{\mu\epsilon'} \left(1 - j \frac{\epsilon''}{\epsilon'}\right)^{1/2} \quad (ii)$$

from (i) & (ii)

Solving these two equations,

$$\alpha = \omega \left\{ \frac{\mu\epsilon'}{2} \left(\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} - 1 \right) \right\}^{1/2} \text{ Np/m} \quad - (8)$$

$$\beta = \omega \left\{ \frac{\mu\epsilon'}{2} \left(\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} + 1 \right) \right\}^{1/2} \text{ rad/m} \quad - (9)$$

We see that a non-zero α (and hence loss) results if the imaginary part of the permittivity ϵ'' , is present.

In eqⁿ (8) & (9) the presence of the ratio ϵ''/ϵ' , which is called the loss tangent.

$$\left| \epsilon' = \frac{\sigma}{\omega} \right.$$