WAVE FRONT →

1. PLANE WAVES → The form of any wave (matter or electromagnetic is determined by its source and described by the shape of its wavefront, i.e., the locus of points of constant phase. If a travelling wave is emitted by a planar source, then the points of constants phase form a plane form a plane surface parallel to the face of the source, Such a wave is called plane wave, and travels in one direction. Since energy is conserved, the total energy in the wave must equal the energy emitted by the source, and therefore the energy density (the energy passing through a unit area), is constant for a plane wave. In a wave of amplitude A and frequency ω, the energy $\varepsilon \propto A^2\omega^2$ Therefore, for a plane wave, the amplitude is constant; the wave does not attenuate.

Plane wave toward $z = +\infty$ at velocity $v_\phi = \frac{\omega}{k}$, wavelength $\lambda = \frac{2\pi}{k}$, frequency $\nu = \frac{\omega}{2\pi}$, amplitude A_0:

$$f[x,y,z,t] = A_0 \cos[kz \mp \omega t]$$

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(n.b; no variation in y or z)

General 3-D plane wave travelling in a direction $k = [k_x, k_y, k_z]$, $r = [x, y, z]$

$$f[r,t] = A_0 \cos[k \cdot r - \omega t]$$

$$\Rightarrow k \cdot r = k_x x + k_y y + k_z z$$

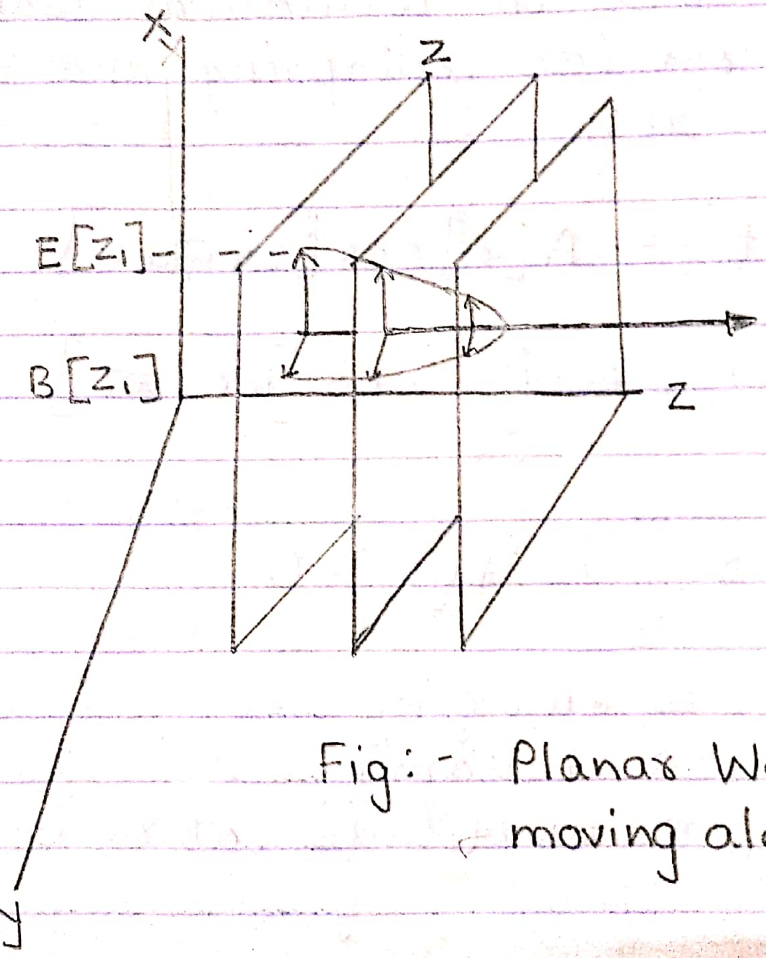

Fig:- Planar Wavefront moving along z-axis

2. <u>CYLINDRICAL WAVES</u> → If a wave is emitted from a line source, the wavefronts are cylindrical. Since the wave expands to fill a cylinder of radius r_0, the wavefront cross is a cylindrical area that grows as Area $= 2\pi r h \propto r$. Therefore, since energy is conserved, the energy per unit area unit

area must decrease as r increases;

$$\frac{\varepsilon}{Area} = constant = \frac{\varepsilon}{2\pi r h} \propto \frac{\varepsilon}{r} \propto \frac{A_0^2}{r} = constant$$

$$\Rightarrow amplitude \propto \frac{A_0}{\sqrt{r}}$$

The equation for a cylindrical wavefront emerging from (or collapsing into) a line source is:

$$f[x,y,z,t] = A[r]\cos[kr \mp \omega t]$$

$$= \frac{A_0}{\sqrt{r}}\cos[kr \mp \omega t]$$

$$r = \sqrt{x^2+y^2} > 0$$

"$-$" $\Rightarrow$ emerging
"$+$" $\Rightarrow$ collapsing
A_0 = amplitude at $r = 0$

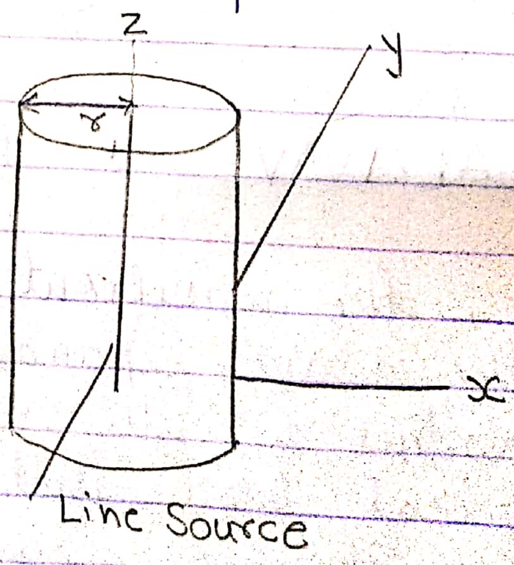

Fig:- Cylindrical waves expanding from a line source

3. SPHERICAL WAVES → The wavefront emerging from (or collapsing into) a point is spherical. The area the wave must cross increases as $x^2+y^2+z^2=r^2$ (area of sphere is $4\pi r^2$). Therefore the energy density drops as r^2 and the amplitude of the wave must decrease as $\frac{1}{r}$. The equation for a spherical wave is

$$f[x,y,z,t] = f[r,t] =$$

$$A[r]\cos[kr \mp \omega t]$$

$$= \frac{A_0}{r}\cos[kr \mp \omega t], \text{ where } r>0$$

"−" ⇒ emerging
"+" ⇒ collapsing
A_0 ⇒ amplitude at $r=0$

Note the pattern for the amplitude of plane, cylindrical, and spherical waves:

plane wave ⇒ 2D- Source (plane)
⇒ amplitude $A[r] \propto r^{-0} = 1$

cylindrical wave ⇒ 1-D Source (line)
⇒ $A[r] \propto r^{-1/2}$

Spherical wave ⇒ 0-D Source (point)
⇒ $A[r] \propto r^{-1}$