Institute of Engineering
Jiwaji University

Unit- IV BE IV Sem

Transmission Line Analogy (Part-II)

EL-402 ELECTRONICS

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Now,

$$\alpha + j\beta = \gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{j^2\omega^2 LC \left(1+\frac{R}{j\omega L}\right)^{\frac{1}{2}} \left(1+\frac{G}{j\omega C}\right)^{\frac{1}{2}}}$$

By taylor Macmillian series

$$(1+x)^{\frac{1}{2}} = 1 + \frac{x}{2} - \frac{x^2}{8}$$

$x << 1$ higher times ignore

Now, seperate real & imaginary

$$\gamma = j\omega\sqrt{LC}\left[1+\frac{R}{2j\omega L}+\frac{R^2}{8\omega^2L^2}\right]\left[1+\frac{G}{2j\omega C}+\frac{G^2}{8\omega^2C^2}\right]$$

$$j\omega\sqrt{LC}\left[1+\frac{G^2}{8\omega^2C^2}-\frac{RG}{G\omega^2LC}+\frac{R^2}{8\omega^2L^2}\right.$$
$$+\frac{R^2G^2}{64\omega^2L^2C^2}+\frac{G}{2j\omega C}+\frac{R}{2j\omega L}$$
$$\left.+\frac{RG^2}{16j\omega^3C^2}+\frac{R^2G}{16\omega^2L^2C}\right]$$

$$\omega\sqrt{LC}\left(\frac{G}{2\omega C}+\frac{R}{2\omega L}+\frac{RG^2}{16\omega^3LC^2}+\frac{R^2G}{16\omega^3L^2C}\right.$$

$$+j\left(1+\frac{G^2}{8\omega^2C^2}-\frac{RG}{G\omega^2LC}+\frac{R^2}{8\omega^2C^2}\right.$$

$$\left.\left.+\frac{R^2G^2}{64\omega^4L^2C^2}\right)\right)$$

$$\omega\sqrt{LC}\left\{\left[\frac{G}{2\omega C}+\frac{R}{2\omega L}\right]+j\left[1+\frac{G^2}{8\omega^2C^2}-\frac{RG}{4\omega^2LC}+\frac{R^2}{8\omega^2L^2}\right]\right\}$$

powers more than 3 are ignored

Real part

$$\alpha = \frac{1}{2}\left(\frac{G\sqrt{LC}}{\sqrt{L}\sqrt{C}} + \frac{R\sqrt{LC}}{\sqrt{L}\sqrt{C}}\right) \quad \Bigg| \quad Z = \sqrt{\frac{L}{C}}\,\Omega$$

$$\alpha = \frac{1}{2}\left(G\sqrt{\frac{L}{C}} + R\sqrt{\frac{C}{L}}\right)$$

$$\beta = \omega\sqrt{LC}\left[1 + \frac{1}{8}\left\{\left(\frac{G}{\omega C}\right)^2 - 2\frac{RG}{\omega^2 LC} + \left(\frac{R}{\omega L}\right)^2\right]$$

$$\beta = \omega\sqrt{LC}\left[1 + \frac{1}{8}\left(-\frac{G}{\omega C} + \frac{R}{\omega L}\right)^2\right]$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

for, loss less transmission

$R = 0, \ G = 0$

$$\gamma = j\omega L + j\omega C$$

$$= 0 + j\omega\sqrt{LC}$$

$$\alpha + j\beta$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{LC}$$

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By dropping in A Re & supressing $e^{j\omega t}$ we transform the voltage to a phasor, which is denoted by an s subcript,

$$V_s(z) = V_0 e^{j\psi} e^{-j\beta z}$$

Now eq^n 1 & 2 can be written as,

Δz — approach zero
ΔI — approach zero

$$\boxed{\frac{dV_s}{dz} = -(R+j\omega L) I_s} \quad -(3)$$

$$\boxed{\frac{dI_s}{dz} = -(G+j\omega C) V_s} \quad -(4)$$

By comparing with Maxwell's curl equations for the uniform plane wave in conducting them in medium.

From

$$\nabla \times E_s = -j\omega\mu H_s$$

$$E_s = E_{xs} a_x$$

$$H_s = H_{ys} a_y$$

we set. $E_s = E_x a_x$ & $H_s = H_{ys} a_y$;

E_{xs} & H_{ys} are functions of z only

& obtain a scalar eq^n we find analogous to ③ & ④

$$\frac{dE_{xs}}{dz} = -j\omega\mu H_{ys}$$

$$\nabla \times H_s = (\sigma + j\omega\varepsilon') E_s$$

$$\frac{dH_{ys}}{dz} = -(\sigma + j\omega\varepsilon') E_{xs}$$

The boundary conditions on V_s & E_{xs} are same there I_s & H_{ys}.

Note :- From it two circuit equations may be obtained from a knowledge of the solution of the two field equations,

$$E_{xs} = E_{xo} e^{-jkz}$$

we obtain voltage equation

$$V_s = V_o e^{-\gamma z}$$

Now, the wave propagates in +z direction with an amplitude

$V = V_0$ at $z=0$, $t=0$ for $y=0$.

The propagation constant for the uniform plane wave,

$$jk = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon')}$$

becomes $\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$

The wavelength $\lambda = \frac{2\pi}{\beta}$

Phase velocity $v_p = \frac{\omega}{\beta}$

& this expression is valid both for the uniform plane wave & transmission line. For a lossless line ($R = G = 0$) we see that

$$\gamma = j\beta = j\omega\sqrt{LC}$$

hence,

$$v_p = 1/\sqrt{LC}$$

From the expression for magnetic field intensity,

$$H_{ys} = \frac{E_{xo}\, e^{-jkz}}{\eta}$$

We see that the positively travelling current wave,

$$I_s = \frac{V_0}{Z_0} e^{-\gamma z}$$

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As equation 1 & 2

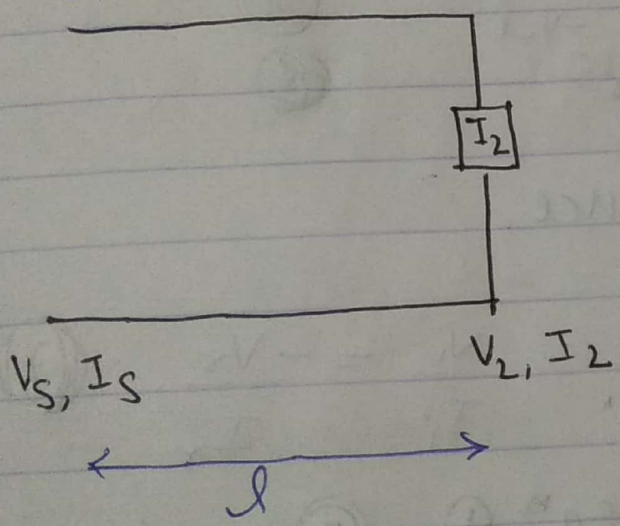
$$\frac{\partial V}{\partial x} = -(R+j\omega L) I = -ZI$$

$$\frac{\partial I}{\partial x} = -(G+j\omega C) V = -yV$$

$$\frac{\partial^2 V}{\partial x^2} = \gamma^2 V, \quad \frac{\partial^2 I}{\partial x^2} = \gamma^2 I$$

$$V = Ae^{-\gamma x} + Be^{+\gamma x}$$
$$I = Ce^{-\gamma x} + De^{+\gamma x}$$

$$V_i(x) = Ae^{-\gamma x}$$
$$I_i(x) = Ce^{-\gamma x}$$



$x = -l$

To find C.F & P.I in this condition only C.F

$$\frac{d^2y}{dx^2} - a^2 y = 0$$

$$m^2 - a^2 = 0$$

$$m = \pm a$$

$$y = C_1 e^{-ax} + C_2 e^{ax}$$

$V_\gamma(x)$, $I_\gamma(x)$

$$V_\gamma(x) = Be^{+\gamma x}$$

$$I_\gamma(x) = De^{+\gamma x}$$

$V_i(0) = A = V_i$ vtg at load
$I_i(0) = C = I_i$
$V_r(0) = B = V_r$
$I_r(0) = D = I_r$

$$\cancel{V(x) = V_i e^{-\gamma x} + V_r e^{+\gamma x}}$$

$$V(x) = V_i e^{-\gamma x} + V_r e^{+\gamma x} \quad (5)$$
$$I(x) = I_i e^{-\gamma x} + I_r e^{+\gamma x} \quad (6)$$

$$V(0) = V_2 = V_i + V_r$$
$$I(0) = I_2 = I_i + I_r$$

$$V(-l) = V_s = V_i e^{\gamma l} + V_r e^{-\gamma l} \quad (7)$$
$$I(-l) = I_s = I_i e^{\gamma l} + I_r e^{-\gamma l} \quad (8)$$

Now, Characteristic impedence,

$$Z_0 = \frac{V_i}{I_i} = \frac{V_i e^{-\gamma x}}{I_i e^{-\gamma x}} = \frac{V_i}{I_i} = -\frac{V_r}{I_r} \quad (9)$$

differentiate eq^n (5), & by Eq^n (1) & (5) equating them

$$-\gamma V_i e^{-\gamma x} + \gamma V_r e^{\gamma x} = -(R + j\omega L) I$$

Putting value from (7)
$$= -Z(I_i e^{-\gamma x} + I_r e^{\gamma x})$$

$$+\gamma Z_0 (I_i e^{-\gamma x} + I_r e^{\gamma x}) = +Z(I_i e^{-\gamma x} + I_r e^{\gamma x})$$

$$Z_0 = \frac{Z}{Y} = \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

$$Z_0 = \frac{\sqrt{R+j\omega L}\,\sqrt{R+j\omega L}}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

$$\boxed{Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$

Loss less transmission

$$R = 0 \quad , \quad G = 0$$

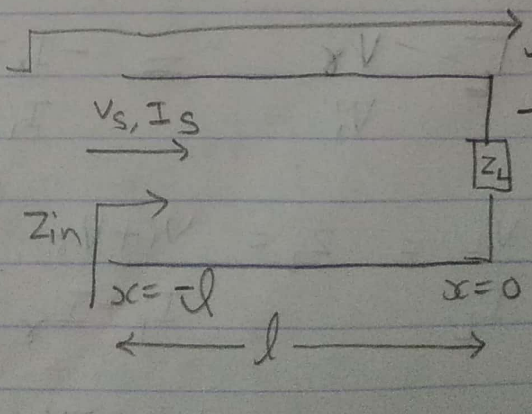
$$Z_0 = \sqrt{\frac{L}{C}}\ \Omega$$

$$\gamma = j\omega\sqrt{LC} = \alpha + j\beta \qquad \Big|\ \alpha = 0$$

$$\beta = \omega\sqrt{LC}$$

$$\gamma = j\beta \qquad \Big|\ A+jB = |\,|\,e^{j\theta}$$

Input Impedence



from -⑦, ⑧ Eqn

$$Z_{in} = \frac{V_s}{I_s} = \frac{V_i e^{\gamma l} + V_r e^{-\gamma l}}{I_i e^{\gamma l} + I_r e^{-\gamma l}}$$

$$= \frac{V_i e^{j\beta l} + V_r e^{-j\beta l}}{I_r e^{j\beta l} + I_r e^{-j\beta l}}$$

$$= \frac{I_i Z_0 e^{j\beta l} - I_r Z_0 e^{-j\beta l}}{I_i e^{j\beta l} + I_r e^{-j\beta l}} = Z_0\left[\frac{I_i e^{j\beta l} - I_r e^{-j\beta l}}{I_i e^{j\beta l} + I_r e^{-j\beta l}}\right]$$

from eqn — ⑨ Eqn

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REFLECTION COEFFICIENT

from eq^n 9

$$\frac{-V_r}{I_r} = \frac{V_i}{I_i} = Z_0 = \sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\Gamma = \frac{V_r}{V_i} \Rightarrow \Gamma(x) = \frac{V_r(x)}{V_i(x)}$$

Γ = Reflection Coefficient

$$\Gamma_L(x) = \frac{V_r(x)}{V_i(x)} = \frac{V_r e^{\gamma x}}{V_i e^{-\gamma x}} = \frac{V_r}{V_i} e^{-2\gamma x}$$

Voltage Reflection Coeffient

As, we know

$$\frac{V_i}{I_i} = Z_0 , \quad \frac{V_r}{I_r} = -Z_0$$

$$\Gamma_L = \frac{V_r}{V_i} = \frac{-I_r Z_0}{I_i Z_0} = \frac{-I_r}{I_i}$$

Current Reflection Coefficient

$$\frac{V_L}{I_L} = Z_L = \frac{V_i + V_r}{I_i + I_r}$$

$$Z_L = \frac{V_i + \Gamma V_i}{I_i - \Gamma I_i} = \frac{V_i}{I_i}\left(\frac{1+\Gamma}{1-\Gamma}\right)$$

$$z_L = \frac{1+\Gamma}{1-\Gamma}$$

$$Z_L - Z_L\Gamma = Z_0 + Z_0\Gamma$$

$$Z_L - Z_0 = (Z_0 + Z_L)\Gamma$$

$$\boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

It is the reflected vtg to incident vtg.
or It is the reflected negative current to incident current.

$$\frac{V_s}{I_s} = \frac{V_i e^{\gamma l} + V_r e^{-\gamma l}}{I_i e^{\gamma l} + I_r e^{-\gamma l}}$$

$$= \frac{V_i e^{j\beta l} + V_r e^{-j\beta l}}{I_i e^{j\beta l} + I_r e^{j\beta l}}$$

$$= \frac{I_i Z_0 e^{j\beta l} - I_r Z_0 e^{-j\beta l}}{I_i e^{j\beta l} + I_r e^{-j\beta l}}$$

$$= Z_0 \left[\frac{I_i e^{j\beta l} - I_r e^{-j\beta l}}{I_i e^{j\beta l} + I_r e^{-j\beta l}}\right]$$

$$= Z_0 \left[\frac{I_i e^{j\beta l} + I_i \Gamma e^{-j\beta l}}{I_i e^{j\beta l} - I_i \Gamma e^{-j\beta l}}\right]$$

$$I_r = -I_i \Gamma$$

$$Z_{in} = Z_0 \left[\frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} \right]$$

$$= Z_0 \left[\frac{e^{j\beta l} + \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j\beta l}}{e^{j\beta l} - \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j\beta l}} \right]$$

$$Z_0 \left[\frac{Z_L(e^{j\beta l} + e^{-j\beta l}) + Z_0(e^{j\beta l} - e^{-j\beta l})}{Z_L(e^{j\beta l} - e^{-j\beta l}) + Z_0(e^{j\beta l} + e^{-j\beta l})} \right]$$

$$Z_{in} = Z_0 \left(\frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} \right)$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$= Z_0 \left(\frac{Z_L 2\cos\theta + Z_0 j 2\sin\theta}{Z_L j 2\sin\theta + Z_0 2\cos\theta} \right)$$

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$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 j \tan\beta l}{Z_0 + Z_L j \tan\beta l} \right]$$

This is in ~~a~~ antenna, the i/p circuit of TV receiver.

Now, charachteristic impedence Z_0 that is analogus to η. Since in conducting meduim.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon'}}$$

We have

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

When a uniform plane wave in meduim 1 is incident on the interface with medium 2,

$$\Gamma = \frac{E_{x0}^-}{E_{x0}^+} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

The fraction of incident wave that is reflected is called the reflection coefficient which the normal incidence is above in Eq^n

Thus the fraction of the incident voltage wave that is reflected by a line with a different characteristic impedence

$$\Gamma = |\Gamma| e^{j\phi} = \frac{V_0^-}{V_0^+} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$

$Z_{01} \rightarrow$ from medium 1

$Z_{02} \rightarrow$ from medium 2

$$Z_{in} = Z_0 \cdot \frac{Z_L \cos\beta l + j Z_0 \sin\beta l}{Z_0 \cos\beta l + j Z_L \sin\beta l}$$

Standing Wave Ratio :-

$$s = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$$I_{max} = V_{max}/Z_0 \quad \% \quad I_{min} = V_{min}/Z_0$$

The input impedence Z_{in} has maxima and minima that occur respectively at the maxima and minima of the voltage and current standing wave.

$$|Z_{in}|_{max} = \frac{V_{max}}{I_{min}} = SZ_0$$

$$|Z_{in}|_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_0}{S}$$

$\eta = \eta_3$ for $z > 0$

$\eta = \eta_2$ for $z < 0$

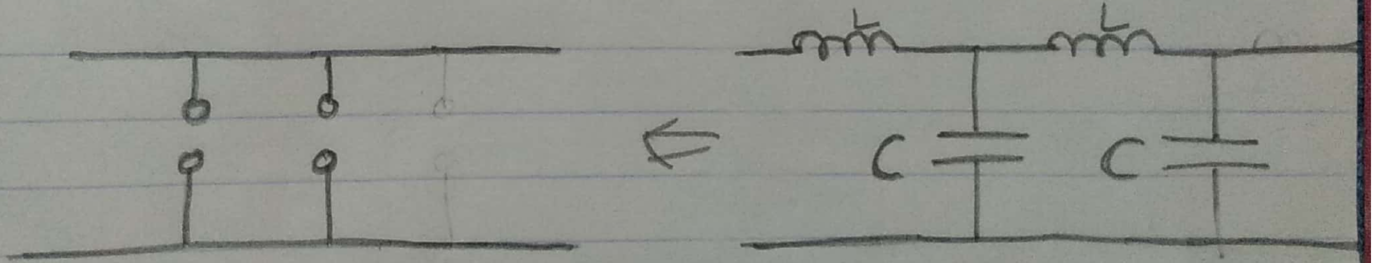
the ratio E_{xs} to H_{ys} at $z = -l$ is

$$\eta_{in} = \eta_2 \frac{\eta_3 \cos\beta_2 l + j\eta_2 \sin\beta_2 l}{\eta_2 \cos\beta_2 l + j\eta_3 \sin\beta_2 l}$$

therefore the input impedence

$$Z_{in} = Z_{02} \frac{Z_{03} \cos\beta_2 l + j Z_{02} \sin\beta_2 l}{Z_{02} \cos\beta_2 l + j Z_{03} \sin\beta_2 l}$$

Note:- Transmission line at low frequency used in power transmission



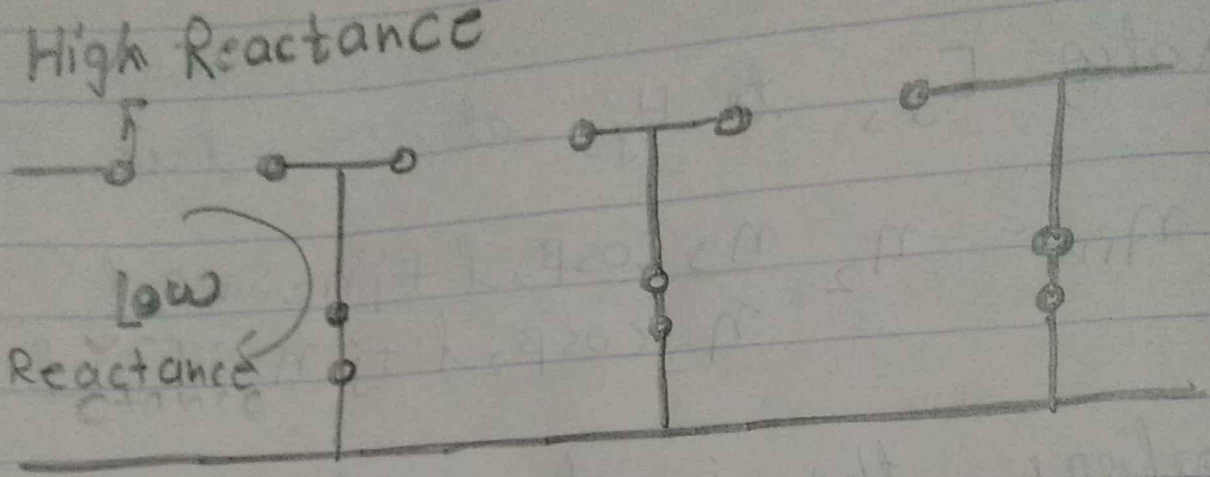
Forward path Short Circuit

i/p → o/p $\quad \omega \to 0 \quad X_L = \omega L \to 0$

$X_C = \frac{1}{\omega C} \to \infty$

By increasing small it goes straight but not reverse.

* Transmission line at higher frequency used in Communication.



$$X_L \rightarrow \omega L \rightarrow \infty$$

$$X_C = \frac{1}{\omega C} \rightarrow 0$$

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