Having found V & A, the fundamental fields are obtained by using gradient,

$$E = -\nabla V \text{ (static)} \quad -(5)$$

or the curl,

$$B = \nabla \times A \text{ (dc)} \quad -(6)$$

By adding Eq^n (5)

$$E = -\nabla V + N$$

taking the curl,

$$\nabla \times E = 0 + \nabla \times N$$

using the point form of Faraday's law,

$$\nabla \times N = -\frac{\partial B}{\partial t}$$

& using (6) giving us,

$$\nabla \times N = -\frac{\partial}{\partial t}(\nabla \times A)$$

or $$\nabla \times N = -\nabla \times \frac{\partial A}{\partial t}$$

& $$E = -\nabla V - \frac{\partial A}{\partial t} \quad -(7)$$

Eq^n 6 & 7 by substituting them into the remaining two of Maxwell's equations:

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\nabla \cdot D = \rho_v$$

We obtain the more complicated expressions

$$\frac{1}{\mu} \nabla \times \nabla \times A = J + \epsilon \left(-\frac{\nabla \partial V}{\partial t} - \frac{\partial^2 A}{\partial t^2} \right)$$

& $$\epsilon \left(-\nabla \cdot \nabla V - \frac{\partial}{\partial t} \nabla \cdot A \right) = \rho_v$$

or

$$\nabla(\nabla \cdot A) - \nabla^2 A = \mu J - \mu\epsilon \left(\frac{\nabla \partial V}{\partial t} + \frac{\partial^2 A}{\partial t^2} \right) \quad \text{— (8)}$$

and

$$\nabla^2 V + \frac{\partial}{\partial t} (\nabla \cdot A) = -\frac{\rho_v}{\epsilon} \quad \text{— (9)}$$

Eq^n 8 & 9 become,

$$\nabla \cdot A = -\mu\epsilon \frac{\partial V}{\partial t} \quad \text{— (10)}$$

$$\nabla^2 A = -\mu J + \mu\epsilon \frac{\partial^2 A}{\partial t^2} \quad \text{— (11)}$$

and $\nabla^2 V = -\frac{\rho_v}{\epsilon} + \mu\epsilon \frac{\partial^2 V}{\partial t^2}$ — (12)

Definition of V & A

$$B = \nabla \times A \quad \text{— (13)}$$

$$\nabla \cdot A = -\mu\epsilon \frac{\partial V}{\partial t} \quad \text{— (14)}$$

$$E = -\nabla V - \frac{\partial A}{\partial t} \quad \text{— (15)}$$

Eq^n (1) becomes,

$$V = \int_{vol} \frac{[\rho_v]}{4\pi\epsilon R} dv \quad \text{— (16)}$$

ρ_v = Every t appearing in the expression for ρ_v has been replaced by a retarted time,

$$t' = t - \frac{R}{v}$$

Thus, if the charge density throughout space were given by,

$$\rho_v = e^{-r} \cos\omega t$$

then

$$[\rho_v] = e^{-r} \cos\left[\omega\left(t - \frac{R}{v}\right)\right]$$

R = Distance between the differential element of charge being considered & the point at which the potential is to be determined.

The retarded vector magnetic potential is given by,

$$A = \int_{Vol} \frac{\mu [J]}{4\pi R} \, dV \qquad -(17)$$

The use of a retarded time has resulted in the time-varying potentials being given the name of retarded potentials.

<u>Note</u>:- If the charge and current distributions are unknown, or reasonable approx. can't made for them, these potentials usually offer no easier path toward the solution than does the direct application of Maxwell's equations.

where a_N is an outward normal at the conductor surface.

RETARTED POTENTIAL →

The time-varying potential usually called retarted potential.

The scalar electric potential V may be expressed in terms of a static charge distribution,

$$V = \int_{vol} \frac{\rho_v \, dv}{4\pi \epsilon R} \quad (static) \qquad -(1)$$

and vector magnetic potential may be found from a current distribution which is constant ~~to~~ with time,

$$A = \int_{vol} \frac{\mu J \, dv}{4\pi R} \quad (dc) \qquad -(2)$$

The differential equations satisfied by V,

$$\nabla^2 V = -\frac{\rho_v}{\epsilon} \quad (static) \qquad -(3)$$

& A

$$\nabla^2 A = -\mu J \quad (dc) \qquad -(4)$$

Eq^n (1) & (2) is integral form,