Institute of Engineering
Jiwaji University

Unit- IV BE IV Sem

PROPAGATION IN GOOD CONDUCTORS

EL-402 EIECTRONICS

EMT

Submitted by:
Swati Dixit
Electronics Dept

PROPAGATION IN GOOD CONDUCTORS:
SKIN EFFECT

Primary transmission of energy must take place in the region outside the conductor, because all time-varying fields attenuate very quickly within a good conductor.

The good conductor has a high conductivity and large conduction currents. The energy represented by the wave travelling through the material therefore decreases as the wave propagates because ohmic losses are continuously present. When we take loss tangent, we saw that the ratio of conduction current density to the displacement current density in a conducting material is given by $\sigma / \omega \varepsilon'$.

The general expression for the propagation constant

$$jk = j\omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon'}}$$

Which simply to obtain,

$$jk = j\omega\sqrt{\mu\varepsilon'}\sqrt{-j\frac{\sigma}{\omega\varepsilon'}}$$

or $\quad jk = j\sqrt{-j\omega\mu\sigma}$ $\qquad$ But $-j = 1\angle -90^\circ$

$\sqrt{1\angle -90^\circ}$

$1\angle -45^\circ = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

Therefore,

$$jk = j\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)\sqrt{\omega\mu\sigma}$$

or $jk = (j1+1)\sqrt{\pi f\mu\sigma} = \alpha + j\beta$

Hence,

$$\alpha = \beta = \sqrt{\pi f\mu\sigma}$$

Regardless of the parameters μ and σ of the conductor or of the frequency of the applied field, $\alpha = \beta$.

If we again assume only an E_x component travelling in the $+z$ direction, then

$$E_x = E_{x0} e^{-z\sqrt{\pi f\mu\sigma}} \cos\left(\omega t - z\sqrt{\pi f\mu\sigma}\right) \quad (1)$$

We may tie this field in the conductor to an external field at the conductor surface.

We let region $z>0$ be the good conductor ~~to an external field at the conductor surface~~, and the region $z<0$ be a perfect dielectric.

At the boundary surface $z = 0$

$$E_x = E_{x0} \cos\omega t \qquad (z=0)$$

We shall consider as the source field that establishes the fields within the conductor. Since the displacement current is negligible.

$$J = \sigma E$$

Thus the conduction current density at any point within the conductor is directly related to E.

$$J_x = \sigma E_x = \sigma E_{x0} e^{-z\sqrt{\pi f \mu \sigma}} \cos(\omega t - z\sqrt{\pi f \mu \sigma}) \quad -(2)$$

Eq^n (1) & (2)

Considering first the negative exponential term, we find an exponential decrease in the conduction current density and electric field intensity with penetration into the conductor (away from the source).

The exponential factor is unity at $z=0$ and decreases to $e^{-1} = 0.368$ when

$$z = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

This distance is denoted by δ and is termed the depth of penetration or the skin depth,

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\alpha} = \frac{1}{\beta}$$

Velocity and Wavelength of good Conductor

$$\alpha = \beta = \frac{1}{\delta} = \sqrt{\pi f \mu \sigma}$$

Then, since $\beta = \frac{2\pi}{\lambda}$

we find the wavelength to be $\lambda = 2\pi\delta$

$$v_p = \frac{\omega}{\beta}$$

$$v_p = \omega\delta$$

We next turn to finding magnetic field, H_y, associated with E_x.

From intrinsic impedance is now a complex quantity,

$$\eta = \sqrt{\frac{\mu}{\epsilon' - j\epsilon''}} = \sqrt{\frac{\mu}{\epsilon'}} \frac{1}{\sqrt{1 - j(\epsilon''/\epsilon')}}$$

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$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon'}} \quad \text{with } \epsilon'' = \frac{\sigma}{\omega}$$

Since $\sigma >> \omega\epsilon'$, we have

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$\eta = \frac{\sqrt{2}\angle 45^\circ}{\sigma\delta} = \frac{1}{\sigma\delta} + j\frac{1}{\sigma\delta}$$

Thus, if we write

$$E_x = E_{x0}\, e^{-z\sqrt{\pi f \mu\sigma}} \cos(\omega t - z\sqrt{\pi f \mu \sigma}$$

in terms of skin depth,

$$E_x = E_{x0}\, e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta}\right)$$

then

$$H_y = \frac{\sigma\delta E_{x0}}{\sqrt{2}}\, e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta} - \frac{\pi}{4}\right)$$

& we see that the maximum amplitude of the magnetic field intensity occurs 1/8 of a cycle later than the max. amplitude of the electric field intensity at every point.

From eqⁿ (3) we may obtain the time-average poynting vector by

$$P_{z,av} = \frac{1}{2} \frac{\sigma \delta E_{x0}^2}{\sqrt{2}} e^{-2z/\delta} \cos\left(\frac{\pi}{4}\right)$$

$$P_{z,av} = \frac{1}{4} \sigma \delta E_{x0}^2 e^{-2\pi/\delta}$$

The total power loss in a width $0<y<b$ and length $0<x<L$ in the direction of the ~~curl~~ current, as shown in Fig 1, is obtained by finding the power crossing the conductor surface within this area,

$$P_{L,av} = \int_S P_{z,av}\, dS$$

$$= \int_0^b \int_0^L \frac{1}{4} \sigma \delta E_{x0}^2 e^{-2z/\delta} \Big|_{z=0} dx\, dy$$

$$= \frac{1}{4} \sigma \delta b E_{x0}^2$$

In terms of the current density J_{x0} at the surface,

$$J_{x0} = \sigma E_{x0}$$

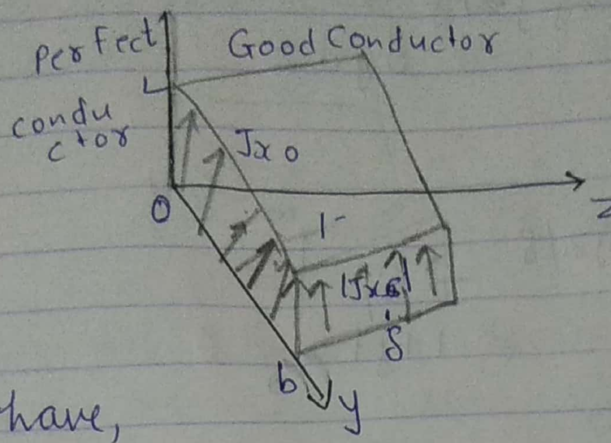

Fig: Showing current density ↓ magnitude as the wave propagates into the conductor.

we have,

$$P_{L,av} = \frac{1}{4\sigma} \delta b L J_{x0}^2$$

Now, let us what power loss would result if the total current in a width b were distributed uniformly in one skin depth. To find the total current, we integrate the current density over the infinite depth of the conductor.

$$I = \int_0^{\infty} \int_0^{b} J_x \, dy \, dz$$

where $J_x = J_{x0} e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta}\right)$

or in complex expontial notation to ~~simply~~ simplify the integration,

$$J_{xs} = J_{x0} e^{-z/\delta} e^{-jz/\delta}$$

$$= J_{x0} e^{-(1+j)z/\delta}$$

Therefore,

$$I_s = \int_0^{\infty}\int_0^{b} J_x e^{-(1+j)z/\delta}\, dy\, dz$$

$$= J_{x0}\, b\, e^{-(1+j1)z/\delta} \left. \frac{-\delta}{1+j1} \right|_0^{\infty}$$

$$= \frac{J_{x0} b \delta}{1+j1}$$

& $$I = \frac{J_{x0} b \delta}{\sqrt{2}} \cos\left(\omega t - \frac{\pi}{4}\right)$$

If this current is distributed with a uniform density J' throughout the cross section $0<y<b$, $0<z<\delta$, then

$$J' = \frac{J_{x0}}{\sqrt{2}} \cos\left(\omega t - \frac{\pi}{4}\right)$$

The ohmic power loss per unit volume is $J \cdot E$, & thus total instantaneous power ~~dissp~~ dissipated in the volume under consideration is

$$P_2 = \frac{1}{\sigma}(J')^2 b L \delta$$

$$= \frac{J_{x0}^2}{2\sigma} b L \delta \cos^2\left(\omega t - \frac{\pi}{4}\right)$$

The time average power loss can be ~~obtained~~ obtained the average value of the cosine-squared factor is one-half.

$$P_{L,av} = \frac{1}{4\sigma} J_{x0}^2 b L \delta$$

Thus the average power loss in a conductor with skin effect present may be calculated by assuming that the total current is distributed uniformly in one skin depth.

The resistance of a width b the length L of an infinitely thick slab with skin ~~length~~ effect is same as the resistance of a rectangular slab of width b, length L, and thickness δ without skin effect, or with uniform current distribution.

We may apply this to a conductor of circular cross section with little error, provided that the radius a is much greater than the skin depth.

The resistance at a high frequency where there is a well-developed skin effect is $\therefore$, a slab of width equal to the circumference $2\pi a$ & thickness δ.

Hence,

$$R = \frac{L}{\sigma \delta} = \frac{L}{2\pi a \sigma \delta}$$

A round copper wire of 1mm radius & 1 km length has a resistance at direct current of

$$R_{dc} = \frac{10^3}{\pi 10^{-6} (5.8 \times 10^7)} = 5.48\,\Omega$$

At 1 MHz, the skin depth is 0.066 mm,

Thus $\delta \ll a$, and the resistance at 1 MHz is found

$$R = \frac{10^3}{2\pi 10^{-3} (5.8 \times 10^7)(0.066 \times 10^{-3})}$$

$$= 41.5\,\Omega$$

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