INSTITUTE OF ENGINEERING, JIWAJI UNIVERSITY

PRODUCTIVITY:
INPUT AND OUTPUT ANALYSIS

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Input-Output Analysis

What Is Input-Output Analysis?

Input-output analysis ("I-O") is a form of macroeconomic analysis based on the interdependencies between economic sectors or industries. This method is commonly used for estimating the impacts of positive or negative economic shocks and analyzing the ripple effects throughout an economy. This type of economic analysis was originally developed by Wassily Leontief (1905–1999), who later won the Nobel Memorial Prize in Economic Sciences for his work in this area.

The foundation of I-O analysis involves input-output tables. Such tables include a series of rows and columns of data that quantify the supply chain for all sectors of an economy. Industries are listed in the headers of each row and each column. The data in each column corresponds to the level of inputs used in that industry's production function.

For example, the column for auto manufacturing shows the resources required for building automobiles (e.g., so much steel, aluminum, plastic, electronics, and so on). I-O models typically include separate tables showing the amount of labor required per dollar unit of investment or production. While input-output analysis is not commonly utilized by neoclassical economics or by policy advisers in the West, it has been employed in Marxist economic analysis of coordinated economies that rely on a central planner.

Three Types of Economic Impact

I-O models estimate three types of impact: direct, indirect, and induced. These terms are another way of referring to initial, secondary and tertiary impacts that ripple throughout the economy when a change is made to a given input level. By using I-O models, economists can estimate the change in output across industries due to a change in inputs in one or more specific industries.

- The direct impact of an economic shock is an initial change in expenditures. For example, building a bridge would require spending on cement, steel, construction equipment, labor, and other inputs.

- The indirect, or secondary, impact would be due to the suppliers of the inputs hiring workers to meet demand.

- The induced, or tertiary, impact would result from the workers of suppliers purchasing more goods and services. This analysis can also be run in reverse, seeing what effects on inputs were likely the cause of observed changes in outputs.

An Example

Here's an example of how I-O analysis works: A local government wants to build a new bridge and needs to justify the cost of the investment. To do so, it hires an economist to conduct an I-O study. The economist talks to engineers and construction companies to estimate how much the bridge will cost, the supplies needed, and how many workers will be hired by the construction company. The economist converts this information into dollar figures and runs numbers through an I-O model, which produces the three levels of impacts. The direct impact is simply the original numbers put into the model, for example, the value of the raw inputs (cement, steel, etc.). The indirect impact is the jobs created by the supplying companies, so cement and steel companies. The induced impact is the amount of money that the new workers spend on goods and services.

Input-Output Analysis in Economics

One of the most interesting developments in the field of modern economics is the model of industrial interdependence known as input-output tableau. It owes its origin to Prof. Wassily Leontief. Input-output analysis is of special interest to the national-income economist because it provides a very detailed breakdown of the macro-aggregates and money flows. This model is widely used in planning and forecasting.

Input-Output Flow Tables:

Leontief imagines an economy in which goods like iron, coal, alcohol, etc. are produced in their respective industries by means of a primary factor, viz., labour, and by means of other inputs such as iron, coal, alcohol, etc. For the production of iron, coal is required.

-Industry Example:

Let us imagine, following Leontief, a simple economy in which there are two industries—agriculture and manufacturing. Each directly requires the use of a primary factor called labour in its production process, and each requires in its productive process inputs which are output of the other industry.

Table 1 provides a simplified picture of such an economy. Agriculture and manufacturing are the first two entries, and each of their rows will show what happens to their total output. The third row is given to the primary factor, labour, of which the community has a total of 50 units (thousands of man-years) per year. These 50 units of labour are allocated as inputs to the two industries in the respective amounts 10 and 40.

The first row total shows that the agricultural output totals 250 units (million of tons) per year. Of this total, 50 units go directly to final consumption, i.e., to households and government, as shown in the third column of row 1. What happens to the remaining 200 units of agricultural output?

They are required as inputs to help make possible the community's production of manufactured and agricultural goods. Thus 175 units of agricultural output is required as material inputs in order to make possible manufacturing production: this is shown in the second column of the first row.

The remainder of agricultural output, 25 units, is required in agriculture itself, e.g., that used to feed cows that turn out wheat, and is shown in column 1 of row h Similarly, row 2 shows the allocation of the total output of manufacturing industries, 120 units (thousand of dozens) per year, among final consumption and intermediate inputs needed in two industries.

In row 2, columns 1, 2 and 3 show allocations of 40, 20 and 60 units of manufactured goods per year to agriculture manufacturing and final consumption (households and governments). All the items in Table 1 are flows, i.e., physical units to per year (and not stocks like capital or intangibles).

Table 1: Inter-Industry Transactions

Industries	Inputs to agriculture	Inputs to manufacturing	Final demand	Total outputs
Agriculture	25	175	50	250
Manufacturing	40	20	60	120
Labour services	10	40	0	50

The 'total outputs' column gives the overall input of labour and output of each commodity. The first column describes the input or cost structure of the agricultural industry : the 250-unit agricultural output was produced with the use of 25 units of agricultural goods, 40 units of manufacturing goods, and 10 units of labour.

Similarly, the second column details the observed input structure of the manufacturing industry. The 'final demand' column shows the commodity breakdown of what is available for consumption and government expenditure. Labour is assumed not to be directly consumed.

Suppose, however, that we had deliberately chosen the physical units in which each commodity is measured so that at some given base prices, one unit costs Re. 1. Then each entry in Table 1 becomes a rupee value and the columns can be measured virtually (literally) as cost figures. If we add down the columns, the sum gives the total cost of producing the industry's output.

Since the output is also measured in terms of rupee values, total output is the same as total revenue. Thus agricultural revenue (at the base prices) is Rs 250 million, and cost of production is Rs 75 mn. In manufacturing, revenue is Rs 120 mn, and cost Rs 235 mn. Thus in agriculture there was a profit of Rs 175 million, and in manufacturing there was a loss of Rs 115 mn.

These items in Table 1 show that the sales of the two industries to themselves and to each other might be described as **“non-GNP”** items. The ‘final demand’ column represents the output side of GNP, and the labour row represents the factor-cost side.
The economy can be thought of as a machine that uses up labour (and has 50 units of labour per year at its disposal) and produces final consumption. With its 50 units of labour the economy is capable of producing an annual flow of 50 units of agricultural goods and 60 units of manufactures.

The Closed Model:

If the exogenous sector of the open input-output model is absorbed into the system as just another industry, the model will become a closed one. In such a model, final demand and primary input do not appear; in their place will be the input requirements and the output of the newly conceived industry. All goods will now be intermediate in nature, because everything is produced only for the sake of satisfying the input requirements of the (n + 1) sectors in the model.

At first glance, the conversion of the open sector into an additional industry would not seem to create any significant change in the analysis. Actually, however, since the new industry is assumed to have a fixed input requirement it must now bear a fixed proportion to the labour service they supply. This constitutes a significant change in the analytical framework of the model.

Mathematically, the disappearance of the final demands means that we will now have a homogeneous equation system.

Assuming four industries only (including the new one, designated by the 0 subscript), the 'correct' output levels will be, by analogy of the above matrix, those which satisfy the following equation systems:

$$\begin{bmatrix} (1-a_{00}) & -a_{01} & -a_{02} & -a_{03} \\ -a_{10} & (1-a_{11}) & -a_{12} & -a_{13} \\ -a_{20} & -a_{21} & (1-a_{22}) & -a_{23} \\ -a_{30} & -a_{31} & -a_{32} & (1-a_{33}) \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Being homogeneous, this equation system can have a non-trivial solution if and only if the 4 x 4 technology matrix (I – A) has a vanishing determinant. The latter condition is indeed always fulfilled : In a closed model, there is no more primary input; hence the column sum in the input-coefficient matrix A must now be exactly equal to (rather than less than) l; that is

$a_{0j} + a_{1j} + a_{2j} + a_{3j} = 1$

or $a_{0j} = 1 – a_{1j} – a_{2j} – a_{3j}$

But this implies that, in every column of the matrix (I-A) above, the top element is always equal to the negative of the sum of the other three elements. Consequently, the four rows are linearly dependent, and we must find |I -A| = 0. This guarantees that the system does possess non-trivial solutions; in fact, it has an infinite number of them.

This means that in a closed model, with a homogeneous-linear equation system, no unique 'correct' output mix exists. We can determine the output levels x_1………………. x_3 in proportion to one another, but cannot fix their absolute levels unless additional restrictions are imposed on the model.

Mathematical Interpretation:

The simple input-output model can well be presented in terms of a few mathematical equations and symbols and on the basis of certain technological assumptions.

If we call agriculture industry 1, manufacturing industry 2 and give labour the subscript 0, then the previous table can be presented as:

Table 4 : An alternative way of presenting Table 3

	Inputs to Industry 1	Inputs to Industry 2	Final consumption	Total output of industries
Industry 1	x_{11}	x_{12}	C_1	X_1
Industry 2	x_{21}	x_{22}	C_2	X_2
Labour services	x_{01}	x_{02}	. . .	X_0

Thus we could write production functions :

$$\left.\begin{aligned} X_1 &= F^1(x_{11}, x_{21}, x_{01}) \\ X_2 &= F^2(x_{12}, x_{22}, x_{02}) \end{aligned}\right\} \quad \ldots(1)$$

since X_1 and X_2 are the total outputs. In addition, we can always add across the rows, so we know that

$$\left.\begin{aligned} x_{11} + x_{12} + C_1 &= X_1 \\ x_{21} + x_{22} + C_2 &= X_2 \\ x_{01} + x_{02} &= X_0 \end{aligned}\right\} \quad \ldots(2)$$

Leontief assumes:

1. There exist constant returns to scale.

2. There exists fixed coefficients of production, i.e., he supposes that it takes a certain minimal input of each commodity per unit of output of each commodity. The word **"minimal"** is of some importance- if it takes 2 tonnes of iron ore to produce 1 tonne of

iron, no doubt the same iron could be produced from even more ore, but as long as iron has value, nobody will be silly enough to use more than the absolutely required 2 tonnes.
This special Leontief production function can be written in the usual form (1). Let a_{1j} be the required minimal input of commodity i per unit of output of commodity j (here i = 0, 1, or 2, and j = 1 or 2).

Then

$$\left.\begin{aligned} X_1 &= \min.\left(\frac{x_{11}}{a_{11}}, \frac{x_{21}}{a_{21}}, \frac{x_{01}}{a_{01}}\right) \\ X_2 &= \min.\left(\frac{x_{12}}{a_{12}}, \frac{x_{22}}{a_{22}}, \frac{x_{02}}{a_{02}}\right) \end{aligned}\right\} \quad \ldots(3) \quad \text{Limitational Production Function}$$

The Leontief system can how be written as :

$$\left.\begin{aligned} X_1 &= a_{11}X_1 + a_{12}X_2 + C_1 \\ X_2 &= a_{21}X_1 + a_{22}X_2 + C_2 \\ L &= a_{01}X_1 + a_{02}X_2 \end{aligned}\right\} \quad \ldots(4)$$

or we can write :

$$\left.\begin{aligned} x_{11} + x_{12} + C_1 &\leq X_1 \\ x_{21} + x_{22} + C_2 &\leq X_2 \\ x_{01} + x_{02} &\leq X_0 \end{aligned}\right\} \quad \ldots(4)'$$

The available output certainly cannot be less than the sum of its alternative uses, but it could, physically, be greater.

We can account for the output X_1 as follows; $a_{11}X_1$ will be used up in industry 1 itself, and $a_{12}X_2$ in industry 2. What is left will be used up for final consumption C_1, viz.,

$C_1 = X_1 - a_{11}X_1 - a_{12}X_2$

Similarly for X_2, i.e., $C_2 = X_2 - a_{21}X_x - a_{22}X_2$. Labour is not produced but is available in amounts up to X_0; the use of labour is $a_{01}X_1$ in industry 1 and $a_{02}X_2$ in industry 2.

Thus we get:

$$\left.\begin{array}{l}(1-a_{11})X_1 - a_{12}X_2 \geq C_1 \\ -a_{21}X_1 + (1-a_{22})X^2 \geq C_2\end{array}\right\} \quad \ldots(5)$$

$$a_{01}X_1 + a_{02}X_2 \leq X_0 \quad \ldots(6)$$

In Fig. 1 the line $(1-a_{11})X_1 - a_{12}X_2 = C_1$ is drawn as L_1.

The slope of L_1 is $\frac{dX_2}{dX_1} = \frac{1-a_{11}}{a_{12}}$ is positive (if $a_{12} \neq 0$).

The line L_2 is where $-a_{21}X_1 + (1-a_{22})X_2 = C_2$ and the slope of L_2 is : $\frac{a_{21}}{1-a_{22}}$

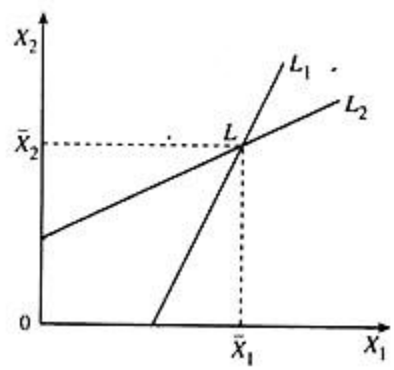

Fig.1: The Hawkins-Simon condition

$L_1$and L_2 intersect at L. If L_1 and L_2 were parallel, i.e., if they had equal slopes, there would be no such point as 1. Any gross-output levels in this region will enable society to consume C_1 and C_2 of the two commodities. In fact, if L_2 had a bigger slope than L_1 there would also be no point L. What is the condition that L should exist or that some bill of goods should be producible? It is that the slope of L_2 must be less than the slope of L_1 i.e.,

$$\frac{a_{21}}{1-a_{22}} < \frac{1-a_{11}}{a_{12}} \quad \ldots(7)$$

$$\text{or } (1-a_{11})(1-a_{22}) - a_{12}a_{21} > 0$$

Another way to write this is in determinant form :

$$\begin{vmatrix} 1-a_{11} & -a_{12} \\ -a_{21} & 1-a_{22} \end{vmatrix} > 0 \quad \ldots(8)$$

As we earlier required that we should not take a direct input of more than one ton of coal to make one tonne of coal, inequalities (7) or (8) assure us that if we add up the direct and indirect inputs of coal that go into a ton of output (coal to make coal) that this will be less than one tonne.

Clearly if a tonne of coal "contains", directly and indirectly, more than a ton of coal, self-contained production is not viable. If a technology is to be viable at all, each of the **"own"** input coefficients, a_{11} and a_{22} must be less than unity. Otherwise, there would be negative net outputs ($1 - a_{11}$ and $1 - a_{22}$).
The inequality (7) together with earlier $1 - a_{11} > 0$ and $1 - a_{22} > 0$ comprise what are called the Hawkins-Simon conditions.
Multiply the first equation in (4) above by $1 - a_{22}$, the second by a_{22} and add to get

$$\overline{X}_1 = \frac{1-a_{22}}{(1-a_{22})(1-a_{11})-a_{12}a_{21}} C_1 + \frac{a_{12}}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} C_2$$

Then multiply the first equation by a_{21}, the second by $1 - a_{11}$, and add to get

$$\overline{X}_2 = \frac{a_{21}}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} C_1 + \frac{1-a_{11}}{(1-a_{11})(1-a_{22})8-a_{12}a_{21}} C_2$$

These two equations show that the outputs are built up linearly out of the final demands C_1 and C_2.

Inequality (6), i.e., $a_{01}X_1 + a_{02}X_2 \leq X_0$ provides us with a gross-output possibility frontier :

$$a_{01}X_1 + a_{02}X_2 = X_0$$

The gross outputs can be expressed as linear functions of the final demands :

$$X_1 = A_{11}C_1 + A_{12}C_2$$

$$X_2 = A_{21}C_1 + A_{22}C_2$$

We can substitute for X_1 and X_2 to get

$$a_{01}(A_{11}C_1 + A_{12}C_2) + a_{02}(A_{21}C_1 + A_{22}C_2) = X_0$$

or, collecting like terms

$$(a_{01}A_{11} + a_{02}A_{21})C_1 + (a_{01}A_{12} + a_{02}A_{22})C_2 = X_0$$

$$\text{or, } A_{01}C_1 + A_{02}C_2 = X_0 \qquad \ldots(9)$$

This, explicitly, is the consumption-possibility frontier.

Here $\frac{X_0}{A_{01}}$ = input-output ratio for commodity 1.

Now A_{01} is the direct labour input not into a unit of C_1 but into the gross direct and indirect X_1 and X_2 needed to support a unit of C_1. In other words, A_{01} represents the total direct and indirect labour embodied in a unit of final consumption of commodity and A_{02} is the same for a unit of final consumption of commodity 2. The schedule in (9) simply says that only those bill of final demand are producible and efficient which require X_0 units of labour to support them.

A consumption possibility schedule (9), drawn in Fig. 2, can be thought of as a social transformation curve. If it is desired to consume only C_1an amount X_0/A_{01} can be produced, given the available resources and technology.

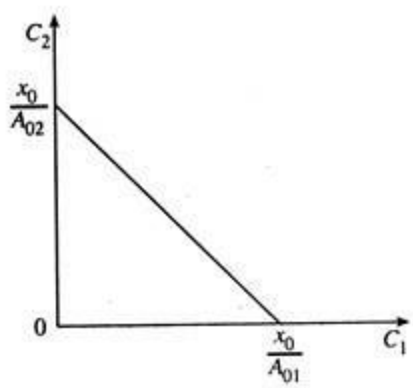

Fig. 2: Consumption Possibility Frontier

If it is desired to give up some C_1in favour of C_2, such substitutions are possible along the transformation curve. Because the frontier is a straight line, substitution of C_2 for C_1 takes place at constant costs. The MRS is constant, viz.,

$$\frac{dC_2}{dC_1} = \frac{A_{01}}{A_{02}}$$

Giving up one unit of C_1 sets free (directly and indirectly) A_0i units of labour. To get 1 more unit of C_2 requires A_{02} units of labour. By giving up 1 unit of C_1 society can, therefore, procure for itself A_{01}/A_{02} units of C_2. The straight line constant cost nature of the transformation curve reflects not only the linearity of the technology, but also the presence of only one primary factor and the absence of joint production.

Prices in the Leontief Model:

The constant MRS was shown to be A_{01}/A_{02}. This must determine the relative price of the two commodities:

$$\frac{P_1}{P_2} = \frac{A_{01}}{A_{02}}$$

We have interpreted A_0i as the total labour content of 1 unit of final output of commodity 1. If we designate the wage rate by W, this tells us that

$$p_1 = A_{01}W = (a_{01}A_{11} + a_{02}A_{21})W$$
$$p_2 = A_{02}W = (a_{01}A_{12} + a_{02}A_{22})W$$

since labour is the only cost-generating element in the system. A real system like Leontief's can only hope to determine relative prices. The absolute level of prices remains completely indeterminate.