CONTINUITY EQUATION →

Continuity Equation can be ~~der~~ derived from Gauss' law and Ampere's law.

The divergence of the curl of any vector field is always zero:

$$\nabla \cdot (\nabla \times H) = 0 \quad -①$$

If we apply the divergence to both sides of Ampere's law, then we obtain:

$$\nabla \cdot \left(\frac{\partial D}{\partial t} + J \right) = \nabla \cdot (\nabla \times H) = 0 \quad -②$$

$$\partial \left(\frac{\nabla \cdot D}{\partial} \right)$$

$$\frac{\partial (\nabla \cdot D)}{\partial t} = -\nabla \cdot J$$

If we apply Gauss' law to the divergence of the Electric flux density (D), we have derived the continuity equation:

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \quad -③$$

Electric current Density (J). This is a measure of whether current is flowing into a volume

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad -(1)$$

$$\nabla \times B = \frac{1}{c^2}\frac{\partial E}{\partial t}$$

exchange among themselves themselves under the replacements

$$E \rightarrow -cB \quad , \quad cB \rightarrow E \quad -(2)$$

Heaviside duality to a continuous transformation:

$$\left.\begin{aligned} E_\eta &= E\cos\eta - cB\sin\eta \\ cB_\eta &= E\sin\eta + cB\cos\eta \end{aligned}\right\}$$

$$0 \leq \eta \leq \frac{\pi}{2} \quad -(3)$$

Maxwell equations have electric charges or currents and/or time varying fields as their sources. Even in vacuum, electric fields are considered the ones accelerating electric charges parallel to their direction, whereas magnetic materials are related to elementary magnetic dipoles but single isolated magnetic charges have never been observed.

Dirac [4], the ~~mq~~ magnetic induction field i.e.

$$\nabla \cdot B = \mu_0 g \delta(x) \qquad (4)$$

To preserve the relation with the magnetic vector potential,

$$B = \nabla \times A \qquad (5)$$

Heaviside duality is a symmetry for the equations of motion but not for the lagrangian providing them. Neglecting sources,

$$L(E, B) = \frac{\varepsilon_0}{2} \int d^3x \left(E^2 - c^2 B^2\right) \qquad (6)$$