SOS in Computer Science And Applications

SOSBCA 401 Numerical Methods

Unit 2nd Jacobi iteration method

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Jacobi's Iteration Method

- The jacobi method is a iterative method of solving the square system of linear equations.
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Jacobi iteration

$$\begin{aligned} a_{11}\boxed{x_1} + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}\boxed{x_2} + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}\boxed{x_n} &= b_n \end{aligned}$$

$$x^0 = \begin{bmatrix} x_1^0 \\ x_2^0 \\ \vdots \\ x_n^0 \end{bmatrix}$$

$$x_1^1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^0 - \cdots - a_{1n}x_n^0)$$

$$x_2^1 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^0 - a_{23}x_3^0 - \cdots - a_{2n}x_n^0)$$

$$x_n^1 = \frac{1}{a_{nn}}(b_n - a_{n1}x_1^0 - a_{n2}x_2^0 - \cdots - a_{nn-1}x_{n-1}^0)$$

$$x_i^{k+1} = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij}x_j^k - \sum_{j=i+1}^{n} a_{ij}x_j^k\right]$$

Jacobi's Iteration Method

- Example:-Solve the system of equations by Jacobi's iteration method.

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

Solution:- We write the equations in the form

$$X = (17 - y + 2z)/20 \quad \text{-----(i)}$$

$$y = (-18 - 3x + z)/20 \quad \text{-----(ii)}$$

$$z = (25 - 2x + 3y)/20 \quad \text{------(iii)}$$

Jacobi's Iteration Method

- We start from an approximation x = y = z = 0.

  Substituting these in the right sides of the equations (i), (ii), (iii), we get

- First iteration:

  x = 0.85, y = -0.9, z = 1.25

- Putting these values again in equations (i), (ii), (iii), we obtain,
- X = [17 – (-0.9) + 2(1.25)] / 20 = 1.02
- Y = [-18 -3(0.85) + 1.25] / 20 = -0.965
- Z = [25 – 2(0.85) + 3(-0.9)] /20 = 1.03

Jacobi's Iteration Method

- Substituting these values again in equations (i), (ii), (iii), we obtain,
- Second iteration:

  $x = 1.00125$, $y = -1.0015$, $z = 1.00325$

- Proceeding in this way, we get,
- Third iteration:

  $x = 1.0004$, $y = -1.000025$, $z = 0.9965$

Jacobi's Iteration Method

- Fourth iteration

  x = 0.999966, y = -1.000078, z = 0.999956

- Fifth iteration

  x = 1.0000, y = -0.999997, z = 0.999992

- The values in the last iterations being practically the same, we can stop.
- Hence the solution is

  x = 1, y = -1, z = 1.