Fermi–Dirac statistics

In quantum statistics, a branch of physics, ***Fermi–Dirac statistics*** describe a distribution of particles over energy states in systems consisting of many identical particles that obey the "Pauli exclusion principle". It is named after Enrico Fermi and Paul Dirac, each of whom discovered the method independently (although Fermi defined the statistics earlier than Dirac).

Fermi–Dirac (F–D) statistics apply to identical particles with half-integer spin in a system with thermodynamic equilibrium. Additionally, the particles in this system are assumed to have negligible mutual interaction. That allows the multi-particle system to be described in terms of single-particle energy states. The result is the F–D distribution of particles over these states which include the condition that no two particles can occupy the same state; this has a considerable effect on the properties of the system. Since ***F–D statistics*** apply to particles with half-integer spin, these particles have come to be called fermions. It is most commonly applied to electrons, a type of fermion with spin 1/2. Fermi–Dirac statistics are a part of the more general field of statistical mechanics and use the principles of quantum mechanics.

The equivalent to F–D statistics is the Bose–Einstein statistics, that apply to bosons (full integer spin, such as photons, or no spin, like the Higgs boson), particles that do not follow the Pauli exclusion principle, meaning that more than one boson can take up the same quantum configuration simultaneously.

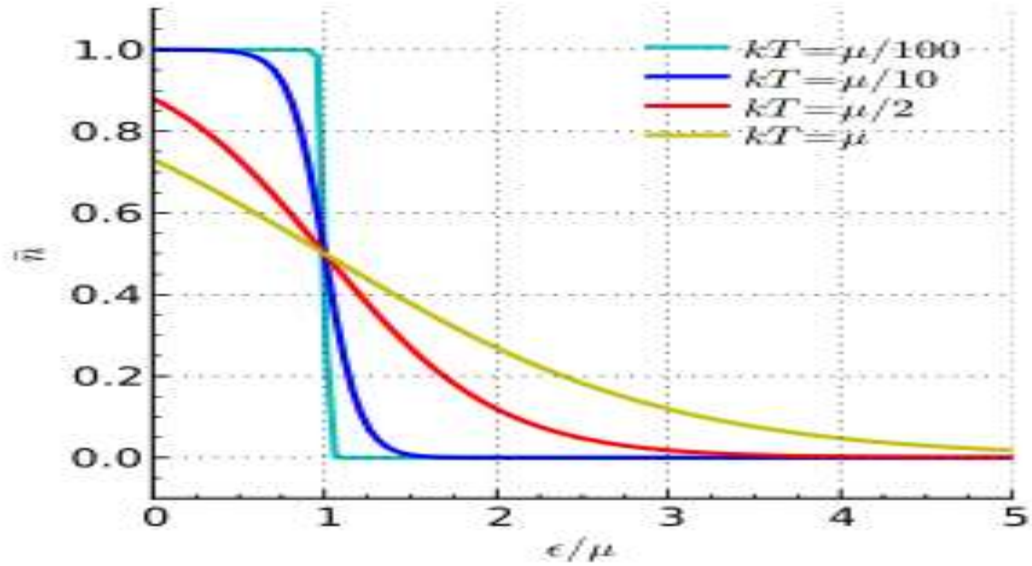
For a system of identical fermions in thermodynamic equilibrium, the average number of fermions in a single-particle state *i* is given by a logistic function, or sigmoid function: the ***Fermi–Dirac (F–D) distribution***, which is a special case of the complete Fermi-Dirac integral,

$$\bar{n}_i = \frac{g_i}{e^{(\varepsilon_i - \mu)/k_B T} + 1}$$

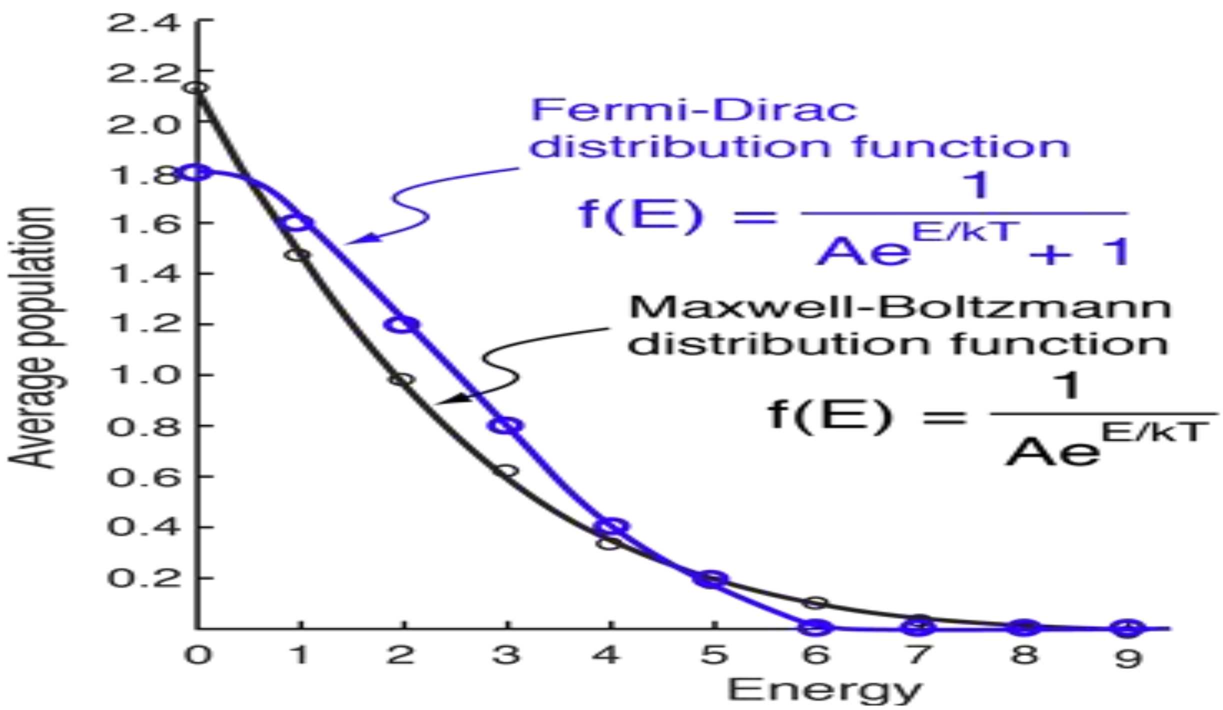
Where, k_B is Boltzmann's constant, T is the absolute temperature, ε_i is the energy of the single-particle state i, and μ is the total chemical potential.

At zero absolute temperature, μ is equal to the Fermi energy plus the potential energy per fermion, provided it is in a neighborhood of positive spectral density. In the case of a spectral gap, such as for electrons in a semiconductor, μ, the point of symmetry, is typically called the Fermi level or—for electrons—the electrochemical potential, and will be located in the middle of the gap.

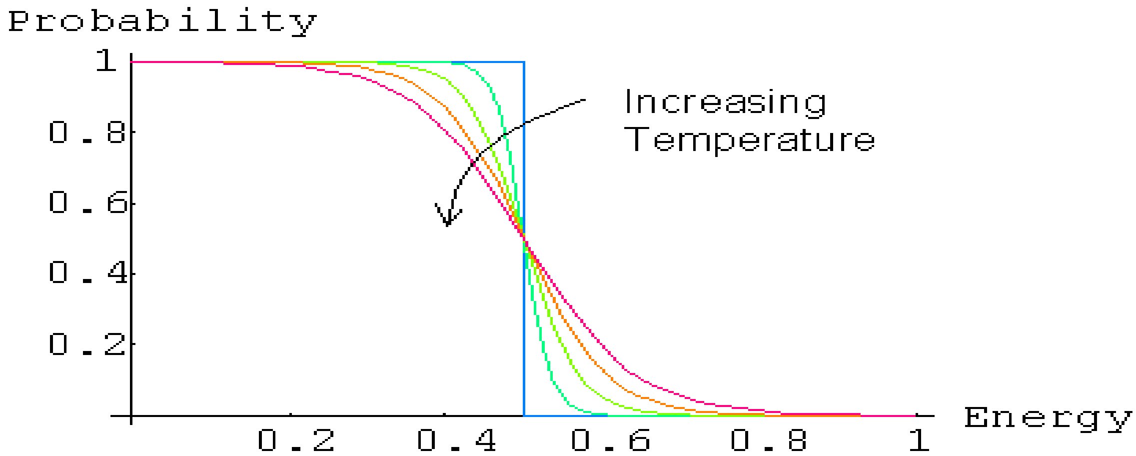
The ***F–D distribution*** is only valid if the number of fermions in the system is large enough so that adding one more fermion to the system has negligible effect on μ. Since the ***F–D distribution*** was derived using the Pauli Exclusion Principle, which allows at most one fermion to occupy each possible state, a result is that $0 < \bar{n}_i < 1$.



1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
$\bar{n}$
ϵ/μ
$kT=\mu/100$
$kT=\mu/10$
$kT=\mu/2$
$kT=\mu$



Average population
Energy
Fermi-Dirac distribution function
$f(E) = \frac{1}{Ae^{E/kT} + 1}$
Maxwell-Boltzmann distribution function
$f(E) = \frac{1}{Ae^{E/kT}}$



Fermi-Dirac Distribution

$$\langle n \rangle = \frac{1}{\exp\left[\left(\varepsilon - \mu\right)/k_B T\right] + 1}$$