

Biostatistics

Z -Test

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Using P -values for a z -Test

The **z -test for the mean** is a statistical test for a population mean. The z -test can be used when the population is normal and σ is known, or for any population when the sample size n is at least 30.

The **test statistic** is the sample mean \bar{x} and the **standardized test statistic** is z .

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \quad \frac{\sigma}{\sqrt{n}} = \text{standard error} = \sigma_{\bar{x}}$$

When $n \geq 30$, the sample standard deviation s can be substituted for σ .

Using P -values for a z -Test

Using P -values for a z -Test for a Mean μ

In Words

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.
2. Specify the level of significance.
3. Determine the standardized test statistic.
4. Find the area that corresponds to z .

In Symbols

State H_0 and H_a .

Identify α .

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Continued.

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In Words

In Symbols

5. Find the P -value.
 - a. For a left-tailed test, $P =$ (Area in left tail).
 - b. For a right-tailed test, $P =$ (Area in right tail).
 - c. For a two-tailed test, $P = 2$ (Area in tail of test statistic).
6. Make a decision to reject or fail to reject the null hypothesis. Reject H_0 if P -value is less than or equal to α . Otherwise, fail to reject H_0 .
7. Interpret the decision in the context of the original claim.

Hypothesis Testing with P -values

Example:

A manufacturer claims that its rechargeable batteries are good for an average of more than 1,000 charges. A random sample of 100 batteries has a mean life of 1002 charges and a standard deviation of 14. Is there enough evidence to support this claim at $\alpha = 0.01$?

$$H_0: \mu \leq 1000$$

$$H_a: \mu > 1000 \quad (\text{Claim})$$

The level of significance is $\alpha = 0.01$.

The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{1002 - 1000}{14 / \sqrt{100}} \\ \approx 1.43$$

Continued.

Hypothesis Testing with P -values

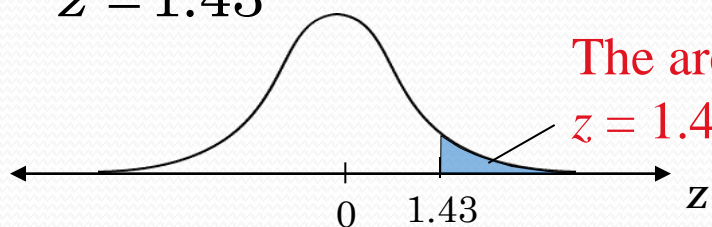
Example continued:

A manufacturer claims that its rechargeable batteries are good for an average of more than 1,000 charges. A random sample of 100 batteries has a mean life of 1002 charges and a standard deviation of 14. Is there enough evidence to support this claim at $\alpha = 0.01$?

$$H_0: \mu \leq 1000$$

$$H_a: \mu > 1000 \quad (\text{Claim})$$

$$z = 1.43$$



The area to the right of $z = 1.43$ is $P = 0.0764$.

P -value is greater than $\alpha = 0.01$, fail to reject H_0 .

At the 1% level of significance, there is not enough evidence to support the claim that the rechargeable battery has an average life of at least 1000 charges.